

Contents

Part I Models and Elementary Mathematics

1	Introduction	3
1.1	Computational Example 1	3
1.2	Computational Example 2	12
1.3	Outline of the Book	15
2	Mathematical Models	19
2.1	*Basic Concepts	19
2.2	*Ordinary Differential Equations	23
2.2.1	Derivatives	23
2.2.2	Differential Equations and Initial Value Problems	27
2.2.3	Integrals	30
2.2.4	More on Differential Equations and Discretization	32
2.3	**Partial Differential Equations	36
2.3.1	Partial Derivatives	36
2.3.2	Boundary and Initial-Boundary Value Problems	41
2.4	**Well Posed Mathematical Models	43
3	Basic Linear Algebra	47
3.1	*Vector Spaces	47
3.2	*Complex Numbers	52
3.3	*Matrix Algebra	54
3.4	**Eigenvalues	62
4	Analysis Tools	67
4.1	*Infinite Processes	67
4.1.1	Infinite Number Sequences	68
4.1.2	Infinite Series	69
4.2	*Taylor Series	72
5	Elementary Functions	77
5.1	*Polynomials	77

5.1.1	Standard Polynomials	77
5.1.2	Piecewise Polynomials	83
5.1.3	Orthogonal Polynomials	89
5.2	*Trigonometric Functions	90
5.3	**The Exponential Function	93

Part II Fundamentals in Numerical Analysis

6	The Fourier Transform	103
6.1	Fourier Series	104
6.2	The Discrete Fourier Transform	109
7	Polynomial Expansions	117
7.1	Orthogonal Polynomial Expansions	117
7.2	Chebyshev Polynomials	118
7.3	Legendre Polynomials	122
8	Least Square Problems	125
8.1	Vector Approximations	125
8.2	The Continuous Case	127
8.3	The Discrete Case	129

Part III Numerical Methods for Differential Equations

9	Numerical Methods for Differential Equations	137
9.1	Basic Principles	137
9.2	Resolution	142
10	Finite Difference Methods	145
10.1	Ordinary Differential Equations	145
10.2	Partial Differential Equations	158
10.3	Fourier Stability Analysis	162
10.4	Several Space Dimensions	167
11	Finite Element Methods	173
11.1	Boundary Value Problems	173
11.2	Initial-Boundary Value Problems	184
11.3	Discontinuous Galerkin Methods	186
12	Spectral Methods	193
12.1	An Example with a Sine Expansion	193
12.2	Fourier Methods	198
12.3	Polynomial Methods	201

Part IV Numerical Methods for Algebraic Equations

13	Numerical Solution of Nonlinear Equations	205
13.1	Scalar Nonlinear Equations	205
13.2	Vector Equations	209

14	Linear Systems of Equations	213
14.1	The Condition of a System	213
14.2	Direct Methods	215
14.3	Iterative Methods	218
14.4	Preconditioning	227
14.5	Multigrid Methods	227
Part V Applications		
15	Wave Propagation	235
15.1	Hyperbolic PDE	235
15.2	Acoustics	240
15.3	Electromagnetics	246
15.4	Other Types of Wave Propagation	252
15.4.1	Elasticity	252
15.4.2	The Schrödinger Equation	253
16	Heat Conduction	255
16.1	Parabolic PDE	255
16.2	Elliptic PDE	258
16.3	The Heat Equation	259
17	Fluid Dynamics	263
17.1	Basic Differential Equations	263
17.1.1	The Continuity Equation	264
17.1.2	Euler equations	265
17.1.3	Shocks	267
17.1.4	Incompressible Navier–Stokes Equations	270
17.2	Low Speed Flow	277
17.3	High Speed Flow and Aircraft Design	280
17.4	Weather Prediction	285
18	Computers and Programming	291
18.1	Basic Principles for Programming	291
18.2	MATLAB	293
18.3	Parallel Computers	300
19	Further Reading	303
Appendix	Mathematical Rules	305
A.1	Rules for Differentiation	305
A.2	Differential Operators in Different Coordinate Systems	306
A.2.1	2D	306
A.2.2	3D	306
A.3	Trigonometric Formulas	307
A.4	Matrix Algebra	308
References		311
Index		313