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VI. THE STOCHASTIC DIFFERENTIAL SYSTEMS

$$\dot{\mathbf{x}}(t; \omega) = \mathbf{A}(\omega) \mathbf{x}(t; \omega) + \mathbf{b}(\omega) \phi(\sigma(t; \omega))$$

WITH

$$\sigma(t; \omega) = \langle \mathbf{c}(t; \omega), \mathbf{x}(t; \omega) \rangle$$

AND

$$\dot{\mathbf{x}}(t; \omega) = \mathbf{A}(\omega) \mathbf{x}(t; \omega) + \mathbf{b}(\omega) \phi(\sigma(t; \omega))$$

WITH

$$\sigma(t; \omega) = \mathbf{f}(t; \omega) + \int_0^t \langle \mathbf{c}(t-\tau; \omega), \mathbf{x}(\tau; \omega) \rangle d\tau130$$

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VII.THE STOCHASTIC DIFFERENTIAL SYSTEMS

$$\dot{x}(t;\omega) = A(\omega)x(t;\omega) + \int_0^t b(t-\eta;\omega) \phi(\sigma(\eta;\omega)) d\eta$$

WITH

$$\sigma(t;\omega) = f(t;\omega) + \int_0^t \langle c(t-\eta;\omega), x(\eta;\omega) \rangle d\eta$$

AND

$$\begin{aligned} \dot{x}(t;\omega) = & A(\omega)x(t;\omega) + \int_0^t b(t-\tau;\omega) \phi(\sigma(\tau;\omega)) d\tau \\ & + \int_0^t c(t-\tau;\omega) \sigma(\tau;\omega) d\tau \end{aligned}$$

WITH

$$\sigma(t;\omega) = f(t;\omega) + \int_0^t \langle d(t-\tau;\omega), x(\tau;\omega) \rangle d\tau \quad \quad .144$$

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VIII.THE STOCHASTIC DIFFERENTIAL SYSTEMS WITH LAG TIME

$$\dot{x}(t;\omega) = A(\omega)x(t;\omega) + B(\omega)x(t-\tau;\omega) + b(\omega) \phi(\sigma(t;\omega))$$

WITH

$$\sigma(t;\omega) = f(t;\omega) + \int_0^t \langle c(t-s;\omega), x(s;\omega) \rangle ds$$

AND

$$\begin{aligned} \dot{x}(t;\omega) = & A(\omega)x(t;\omega) + B(\omega)x(t-\tau;\omega) \\ & + \int_0^t \eta(t-u;\omega) \phi(\sigma(u;\omega)) du + b(\omega) \phi(\sigma(t;\omega)) \end{aligned}$$

WITH

$$\sigma(t;\omega) = f(t;\omega) + \int_0^t \langle c(u;\omega), x(t-u;\omega) \rangle du \quad \quad .156$$

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