

V.I. Arnold V.V. Kozlov
A.I. Neishtadt

Mathematical Aspects of Classical and Celestial Mechanics

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Mathematical Aspects of Classical and Celestial Mechanics

V.I. Arnold V.V. Kozlov A.I. Neishtadt

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