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Elements of Queueing Theory

Palm Martingale Calculus
and Stochastic Recurrences

Second Edition



Springer

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