

Contents

Preface	v
0 Introduction	1
0.1 List of equations	1
0.1.1 One-dimensional pseudoparabolic equations	1
0.1.2 One-dimensional wave dispersive equations	2
0.1.3 Singular one-dimensional pseudoparabolic equations	3
0.1.4 Multidimensional pseudoparabolic equations	3
0.1.5 New nonlinear pseudoparabolic equations with sources	5
0.1.6 Model nonlinear equations of even order	6
0.1.7 Multidimensional even-order equations	7
0.1.8 Results and methods of proving theorems on the nonexistence and blow-up of solutions for pseudoparabolic equations	10
0.2 Structure of the monograph	13
0.3 Notation	14
1 Nonlinear model equations of Sobolev type	20
1.1 Mathematical models of quasi-stationary processes in crystalline semi- conductors	20
1.2 Model pseudoparabolic equations	27
1.2.1 Nonlinear waves of Rossby type or drift modes in plasma and appropriate dissipative equations	27
1.2.2 Nonlinear waves of Oskolkov–Benjamin–Bona–Mahony type	29
1.2.3 Models of anisotropic semiconductors	34
1.2.4 Nonlinear singular equations of Sobolev type	37
1.2.5 Pseudoparabolic equations with a nonlinear operator on time derivative	38
1.2.6 Nonlinear nonlocal equations	39
1.2.7 Boundary-value problems for elliptic equations with pseudoparabolic boundary conditions	46
1.3 Disruption of semiconductors as the blow-up of solutions	48
1.4 Appearance and propagation of electric domains in semiconductors	56
1.5 Mathematical models of quasi-stationary processes in crystalline elec- tromagnetic media with spatial dispersion	60

1.6	Model pseudoparabolic equations in electric media with spatial dispersion	64
1.7	Model pseudoparabolic equations in magnetic media with spatial dispersion	66
2	Blow-up of solutions of nonlinear equations of Sobolev type	69
2.1	Formulation of problems	69
2.2	Preliminary definitions, conditions, and auxiliary lemmas	70
2.3	Unique solvability of problem (2.1) in the weak generalized sense and blow-up of its solutions	78
2.4	Unique solvability of problem (2.1) in the strong generalized sense and blow-up of its solutions	101
2.5	Unique solvability of problem (2.2) in the weak generalized sense and estimates of time and rate of the blow-up of its solutions	111
2.6	Strong solvability of problem (2.2) in the case where $\mathbb{B} \equiv 0$	127
2.7	Examples	133
2.8	Initial-boundary-value problem for a nonlinear equation with double nonlinearity of type (2.1)	141
2.8.1	Local solvability of problem (2.131)–(2.133) in the weak generalized sense	142
2.8.2	Blow-up of solutions	159
2.9	Problem for a strongly nonlinear equation of type (2.2) with inferior nonlinearity	164
2.9.1	Unique weak solvability of problem (2.185)	165
2.9.2	Solvability in a finite cylinder and blow-up for a finite time	177
2.9.3	Rate of the blow-up of solutions	183
2.10	Problem for a semilinear equation of the form (2.2)	187
2.10.1	Blow-up of classical solutions	187
2.11	On sufficient conditions of the blow-up of solutions of the Boussinesq equation with sources and nonlinear dissipation	196
2.11.1	Local solvability of strong generalized solutions	197
2.11.2	Blow-up of solutions	200
2.12	Sufficient conditions of the blow-up of solutions of initial-boundary-value problems for a strongly nonlinear pseudoparabolic equation of Rosenau type	203
2.12.1	Local solvability of the problem in the strong generalized sense	203
2.12.2	Blow-up of strong solutions of problem (2.288)–(2.289) and solvability in any finite cylinder	211
2.12.3	Physical interpretation	215

3 Blow-up of solutions of strongly nonlinear Sobolev-type wave equations and equations with linear dissipation	216
3.1 Formulation of problems	216
3.2 Preliminary definitions and conditions and auxiliary lemma	217
3.3 Unique solvability of problem (3.1) in the weak generalized sense and blow-up of its solutions	219
3.4 Unique solvability of problem (3.1) in the strong generalized sense and blow-up of its solutions	244
3.5 Unique solvability of problem (3.2) in the weak generalized sense and blow-up of its solutions	254
3.6 Unique solvability of problem (3.2) in the strong generalized sense and blow-up of its solutions	273
3.7 Examples	278
3.8 On certain initial-boundary-value problems for quasilinear wave equations of the form (3.2)	288
3.8.1 Local solvability in the strong generalized sense of problems (3.141)–(3.143)	288
3.8.2 Blow-up of solutions	295
3.8.3 Breakdown of weakened solutions of problem (3.141)	302
3.9 On an initial-boundary-value problem for a strongly nonlinear equation of the type (3.1) (generalized Boussinesq equation)	308
3.9.1 Unique solvability of the problem in the weak sense	309
3.9.2 Blow-up of solutions and the global solvability of the problem	315
3.10 Blow-up of solutions of a class of quasilinear wave dissipative pseudoparabolic equations with sources	320
3.10.1 Unique local solvability of the problem in the strong sense and blow-up of its solutions	320
3.10.2 Examples	327
3.11 Blow-up of solutions of the Oskolkov–Benjamin–Bona–Mahony–Burgers equation with a cubic source	329
3.11.1 Unique local solvability of the problem	330
3.11.2 Global solvability and the blow-up of solutions	333
3.11.3 Physical interpretation of the obtained results	337
3.12 On generalized Benjamin–Bona–Mahony–Burgers equation with pseudo-Laplacian	337
3.12.1 Blow-up of strong generalized solutions	337
3.12.2 Physical interpretation of the obtained results	340
3.13 Sufficient, close to necessary, conditions of the blow-up of solutions of one problem with pseudo-Laplacian	341
3.13.1 Blow-up of strong generalized solutions	341
3.13.2 Physical interpretation of the obtained results	345

3.14	Sufficient, close to necessary, conditions of the blow-up of solutions of strongly nonlinear generalized Boussinesq equation	345
4	Blow-up of solutions of strongly nonlinear, dissipative wave Sobolev-type equations with sources	357
4.1	Introduction. Statement of problem	357
4.2	Unique solvability of problem (4.1) in the weak generalized sense and blow-up of its solutions	358
4.3	Unique solvability of problem (4.1) in the strong generalized sense and blow-up of its solutions	380
4.4	Examples	385
4.5	Blow-up of solutions of a Sobolev-type wave equation with nonlocal sources	391
4.5.1	Unique local solvability of the problem	391
4.5.2	Blow-up of strong generalized solutions	398
4.6	Blow-up of solutions of a strongly nonlinear equation of spin waves	402
4.6.1	Unique local solvability in the strong generalized sense	403
4.6.2	Blow-up of strong generalized solutions and the global solvability	412
4.6.3	Physical interpretation of the obtained results	417
4.7	Blow-up of solutions of an initial-boundary-value problem for a strongly nonlinear, dissipative equation of the form (4.1)	417
4.7.1	Local unique solvability in the weak generalized sense	418
4.7.2	Unique solvability of the problem and blow-up of its solution for a finite time	435
5	Special problems for nonlinear equations of Sobolev type	439
5.1	Nonlinear nonlocal pseudoparabolic equations	439
5.1.1	Global-on-time solvability of the problem	439
5.1.2	Global-on-time solvability of the problem in the strong generalized sense in the case $q \geq 1$	469
5.1.3	Asymptotic behavior of solutions of problem (5.1), (5.2) as $t \rightarrow +\infty$ in the case $q > 0$	471
5.2	Blow-up of solutions of nonlinear pseudoparabolic equations with sources of the pseudo-Laplacian type	475
5.2.1	Blow-up of weakened solutions of problem (5.77)	476
5.2.2	Blow-up and the global-on-time solvability of problem (5.78)	477
5.2.3	Blow-up of solutions of problem (5.79)	479
5.2.4	Blow-up of weakened solutions of problems (5.80) and (5.81)	482
5.2.5	Interpretation of the obtained results	484

5.3	Blow-up of solutions of pseudoparabolic equations with fast increasing nonlinearities	484
5.3.1	Local solvability and blow-up for a finite time of solutions of problems (5.112) and (5.113)	485
5.3.2	Local solvability and blow-up for a finite time of solutions of problem (5.114)	492
5.4	Blow-up of solutions of nonhomogeneous nonlinear pseudoparabolic equations	496
5.4.1	Unique local solvability of the problem	496
5.4.2	Blow-up of strong generalized solutions of problem (5.154)–(5.155)	499
5.4.3	Blow-up of classical solutions of problem (5.154)–(5.155)	502
5.5	Blow-up of solutions of a nonlinear nonlocal pseudoparabolic equation	503
5.5.1	Unique local solvability of the problem	504
5.5.2	Blow-up and global solvability of problem (5.177)	506
5.5.3	Blow-up rate for problem (5.177) under the condition $q = 0$	509
5.6	Existence of solutions of the Laplace equation with nonlinear dynamic boundary conditions	511
5.6.1	Reduction the problem to the system of the integral equations	511
5.6.2	Global-on-time solvability and the blow-up of solutions	517
5.7	Conditions of the global-on-time solvability of the Cauchy problem for a semilinear pseudoparabolic equation	525
5.7.1	Reduction of the problem to an integral equation	525
5.7.2	Theorems on the existence/nonexistence of global-on-time solutions of the integral equation (5.219)	527
5.8	Sufficient conditions of the blow-up of solutions of the Boussinesq equation with nonlinear Neumann boundary condition	537
6	Numerical methods of solution of initial-boundary-value problems for Sobolev-type equations	543
6.1	Numerical solution of problems for linear equations	543
6.1.1	Dynamic potentials for one equation	544
6.1.2	Solvability of Dirichlet problem	548
6.2	Numerical method of solving initial-boundary-value problems for nonlinear pseudoparabolic equations by the Rosenbrock schemes	554
6.2.1	Stiff method of lines	554
6.2.2	Stiff systems of ODE and methods of solving them	555
6.2.3	Stiff stability	555
6.2.4	Schemes of Rosenbrock type	555
6.2.5	ε -embedding method	557

6.3	Results of blow-up numerical simulation	560
6.3.1	Blow-up of pseudoparabolic equations with a linear operator by the time derivative	561
6.3.2	Blow-up of strongly nonlinear pseudoparabolic equations . . .	566
6.3.3	Blow-up of equations with nonlocal terms (coefficients of the equation depend on the norm of the function)	575
Appendix A Some facts of functional analysis		581
A.1	Sobolev spaces $W^{s,p}(\Omega)$, $W_0^{s,p}(\Omega)$, and $W^{s,p}(\Gamma)$	581
A.2	Weak and $*$ -weak convergence	583
A.3	Weak and strong measurability. Bochner integral	584
A.4	Spaces of integrable functions and distributions	585
A.5	Nemytskii operator. Krasnoselskii theorem	586
A.6	Inequalities	588
A.7	Operator calculus	589
A.8	Fixed-point theorems	589
A.9	Weakened solutions of the Poisson equation	589
A.10	Intersections and sums of Banach spaces	591
A.11	Classical, weakened, strong generalized, and weak generalized solu- tions of evolutionary problems	592
A.12	Two equivalent formulations of weak solutions in $L^2(0, T; \mathbb{B})$	594
A.13	Gâteaux and Fréchet derivatives of nonlinear operators	596
A.14	On the gradient of a functional	604
A.15	Lions compactness lemma	606
A.16	Browder–Minty theorem	607
A.17	Sufficient conditions of the independence of the interval, on which a solution of a system of differential equations exists, of the order of this system	608
A.18	On the continuity of some inverse matrices	610
Appendix B To Chapter 6		613
B.1	Convergence of the ε -embedding method with the CROS scheme . . .	613
Bibliography		621
Index		647