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Fundamentals of Crystals

Symmetry, and Methods of Structural Crystallography

Second, Enlarged Edition

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Contents

I.	Crys	stalline	State	
	1.1	Macro	oscopic Characteristics of Crystals	1
		1.1.1	Crystals and Crystalline Matter	1
		1.1.2	Homogeneity of a Crystalline Substance	5
		1.1.3	Anisotropy of a Crystalline Substance	6
		1.1.4	Symmetry	9
		1.1.5	Crystal Habit	11
	1.2	Micro	structure of a Crystalline Substance	12
		1.2.1	Space Lattice	12
		1.2.2	Experimental Evidence for the Existence of the Crystal	
			Lattice	15
		1.2.3	Reasons for the Microperiodicity Principle	18
	1.3	Struct	tures with Distortions of the Three-Dimensional	
		Period	dicity. Quasicrystals	23
	1.4	Struct	ural Characteristics of Condensed Phases	24
2.	Fun	lamentals of the Theory of Symmetry		
	2.1	The C	Concept of Symmetry	28
		2.1.1	Definition of Symmetry	28
		2.1.2	Symmetry Operations	29
	2.2	Space	Transformations	31
		2.2.1	Space, an Object in It, Points of Space	31
		2.2.2	Basic Isometric Transformations of Space	32
		2.2.3	Analytical Expression for Symmetry Transformations	39
		2.2.4	Relationships and Differences Between Operations of	
			the First and Second Kind	41
	2.3	Funda	amentals of Group Theory	44
		2.3.1	Interaction of Operations	44
		2.3.2	Group Axioms	44
		2.3.3	Principal Properties of Groups	46
		2.3.4	Cyclic Groups, Generators	47
		2.3.5	Subgroup	48
		2.3.6	Cosets, Conjugates, Classes, Expansion with Respect	
			to a Subgroup	48
		2.3.7	Group Products	49
		2.3.8	Group Representations	51

2.4		of Symmetry Groups and Their Properties	53
	2.4.1	Homogeneity, Inhomogeneity, and Discreteness	
		of Space	53
	2.4.2	Types of Symmetry Groups and Their Periodicity	55
	2.4.3	One-Dimensional Groups G^1	57
	2.4.4	Two-Dimensional Groups G^2	58
	2.4.5	Crystallographic Groups	60
	2.4.6	Three-Dimensional Groups G^3	61
2.5	Geom	etric Properties of Symmetry Groups	64
	2.5.1	Symmetry Elements	64
	2.5.2	Summary and Nomenclature of Symmetry Elements	66
	2.5.3	Polarity	71
	2.5.4	Regular Point Systems	71
	2.5.5	Independent Region	73
	2.5.6	Description of a Symmetric Object by Groups of	
		Permutations	77
	2.5.7	Enantiomorphism	79
2.6	Point	Symmetry Groups	82
	2.6.1	Description and Representation of Point Groups	82
	2.6.2	On Derivation of Three-Dimensional	
		Point Groups G_0^3	84
	2.6.3	Point Group Families	86
	2.6.4	Classification of Point Groups	95
	2.6.5	Isomorphism of Groups K	100
	2.6.6	Representations of Point Groups K	101
	2.6.7	Group Representations and Proper Functions	106
2.7		netry Groups G_1^2 , G_2^2 , G_1^3 , G_2^3	107
	2.7.1	Symmetry Groups G_1^2 of Borders	107
	2.7.2	Plane Twice-Periodic Groups G_2^2	108
	2.7.3	Cylindrical (Helical) Groups G_1^2	110
	2.7.4	Layer Groups G_2^3	116
2.8		Groups of Symmetry	120
	2.8.1	Three-Dimensional Lattice	120
	2.8.2	Syngonies	122
	2.8.3	Bravais Groups	123
	2.8.4	Homomorphism of Space and Point Groups	129
	2.8.5	Geometric Rules for Performing Operations	12
	2.0.5	and for Mutual Orientation of Symmetry Elements	
		in Groups Φ	130
	2.8.6	Principles of Derivation of Space Groups.	150
	2.0.0	Symmorphous Groups $\Phi_{\rm s}$	131
	2.8.7	Nonsymmorphous Groups Φ_n	135
	2.8.7	Number of Fedorov Groups	138
	2.8.9	Nomenclature of Fedorov Groups	139
	2.8.10		144
	2.0.10	Buogroups of redotor Groups	144

			Contents	XVII
		2811	Regular Point Systems of Space Groups	145
			Relationship Between the Chemical Formula of a	143
		2.0.12	Crystal and Its Space Symmetry	146
		2.8.13	Local Condition of Space Symmetry	147
			Division of Space	149
			Irreducible Representations of Groups Φ	155
	2.9		ralized Symmetry	156
		2.9.1	On the Extension of the Symmetry Concept	156
		2.9.2	Antisymmetry and Color Symmetry	157
		2.9.3	Antisymmetry Point Groups	159
		2.9.4	Point Groups of Color Symmetry	168
		2.9.5	Space and Other Groups of Antisymmetry and	
			Color Symmetry	171
		2.9.6	Symmetry of Similarity	175
		2.9.7	Partial Symmetry	177
		2.9.8	Statistical Symmetry. Groupoids	177
3.			of the Crystalline Polyhedron and Lattice	
	3.1		Laws of Geometric Crystallography	179
		3.1.1	Law of Constancy of Angles	179
		3.1.2	Law of Rational Parameters. Lattice	180
	3.2		alline Polyhedron	182
		3.2.1	Ideal Shape. Bundle of Normals and Edges	182
		3.2.2	Simple Forms	183
		3.2.3	Distribution of Simple Forms Among Classes	189
		3.2.4	Holohedry and Hemihedry	192
		3.2.5 3.2.6	Combinations of Simple Forms	192
	2 2			193 194
	3.3	3.3.1	Ometry	194
		3.3.2	Crystal Setting	194
		3.3.3	Goniometric Calculations	200
	3.4		ee Geometry	205
	3.4	3.4.1	Straight Lines and Planes of the Lattice	205
		3.4.2	Properties of Planes	206
		3.4.3	Reciprocal Lattice	208
	3.5		the Transformations	212
	5.5	3.5.1	Transformation of Coordinates and Indices in the	212
		0.0.1	Atomic and Reciprocal Lattices	212
		3.5.2	Reduction Algorithm	216
		3.5.3	Computation of Angles and Distances in Crystals	219
			1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	
4.	Stru	cture A	Analysis of Crystals	
-	4.1		amentals of Diffraction Theory	222
			Wave Interference	222

	4.1.2	Scattering Amplitude	224
	4.1.3	Electron Density Distribution. Fourier Integral	226
	4.1.4	Atomic Amplitude	227
	4.1.5	The Temperature Factor	231
4.2	Diffra	ction from Crystals	234
	4.2.1	Laue Conditions. Reciprocal Lattice	234
	4.2.2	Size of Reciprocal Lattice Nodes	237
	4.2.3	Reflection Sphere	239
	4.2.4	Structure Amplitude	241
	4.2.5		242
		Intensity of Reflections	
	4.2.6	Thermal Diffusion Scattering	244
	4.2.7	Symmetry of the Diffraction Pattern and Its	
		Relation to the Point Symmetry of the Crystal	244
	4.2.8	Manifestation of Space-Symmetry of a Crystal in a	
		Diffraction Pattern. Extinctions	245
4.3	Intens	sity of Scattering by a Single Crystal. Kinematic	
	and I	Dynamic Theories	252
	4.3.1	Kinematic Theory	252
	4.3.2	Integrated Intensity of Reflection in Kinematic	
		Scattering	253
	4.3.3	Principles of Dynamic Theory	256
	4.3.4	Darwin's Treatment	257
	4.3.5	Laue-Ewald Treatment	258
	4.3.6	Dynamic Scattering in an Absorbing Crystal.	
		Borrmann Effect	262
	4.3.7	Experimental Investigations and Applications	
	1.5.7	of Dynamic Scattering	266
4.4.	Scatt	ering by Noncrystalline Substances	271
7.7.	4.4.1	General Expression for Intensity of Scattering.	2/1
	4.4.1	Function of Interatomic Distances	271
	4.4.2		2/1
	4.4.2	Spherically Symmetric Systems: Gas, Liquid,	272
		and Amorphous Substances	272
	4.4.3	Systems with Cylindrical Symmetry: Polymers	27.4
		and Liquid Crystals	274
	4.4.4	Small-Angle Scattering	276
4.5	_	rimental Technique of X-Ray Structure Analysis of	
		e Crystals	278
	4.5.1	Generation and Properties of X-Rays	278
	4.5.2	Interaction of X-Rays with a Substance	282
	4.5.3	Recording of X-Rays	283
	4.5.4	Stages of X-Ray Structure Analysis of Single Crystals	284
	4.5.5	Laue Method	285
	4.5.6	Crystal Rotation and Oscillation Methods	287
	4.5.7	Moving Crystal and Film Techniques	291
	7.5.7	1110 this Cijotal and I him I conniques	/ 1

	4.5.8	X-Ray Diffractometers for Investigating Single	
		Crystals	295
	4.5.9	Diffractometric Determination of the Crystal Orientation,	
		Unit Cell, and Intensities	297
4.6		Investigation of Polycrystalline Materials	299
	4.6.1	Potentialities of the Method	299
	4.6.2	Cameras for Polycrystalline Specimens	300
	4.6.3	Indexing of Debye Photographs and Intensity of	
		Their Lines	303
	4.6.4	Diffractometry of Polycrystalline Specimens	304
	4.6.5	Phase Analysis	305
	4.6.6	Investigation of Textures	305
	4.6.7	Determination of the Sizes of Crystals and Internal	207
	ъ.	Stresses	307
4.7		mination of the Atomic Structure of Crystals	307
	4.7.1	Preliminary Data on the Structure	307
	4.7.2	Fourier Synthesis. Phase Problem	308
	4.7.3	The Trial and Error Method. Reliability Factor	312
	4.7.4	The Patterson Interatomic-Distance Function	313
	4.7.5	Heavy-Atom Method	319
	4.7.6	Direct Methods	321
	4.7.7	"Statistical-Thermodynamical" Approach to the	
		Crystal Structure Determination	325
	4.7.8	Nonlocal-Search Method	327
	4.7.9	Determination of the Absolute Configuration	330
	4.7.10	Structure Refinement	331
	4.7.11	Difference Fourier Syntheses	332
	4.7.12	Automation of the Structure Analysis	334
4.8	Electr	on Diffraction	335
	4.8.1	Features of the Method	335
	4.8.2	Experimental Technique	336
	4.8.3	Electron Diffraction Structure Analysis	338
	4.8.4	Dynamic Scattering of Electrons	349
	4.8.5	Special Methods of Electron Diffraction	350
	4.8.6	Low-Energy Electron Diffraction (LEED)	355
4.9		on Microscopy	357
	4.9.1	The Features of the Methods	357
	4.9.2	Transmission Electron Microscopy	358
	4.9.3	HREM Imaging of Atomic Structure of Crystals	368
	4.9.4	EM in Molecular Biology. Experimental Technique	374
	4.9.4	Processing of the Images of Biomolecules.	J 1 -T
	4.7.3	3D Reconstruction	376
	4.9.6	Two-Dimensional Biocrystals	381
		TEM of Single Bioparticles	382
	4.9.7	LEIVE OF SINGLE DIOPARTICLES	302

			Scanning Electron Microscopy (SEM) of Solids	383			
	4.10	Scann	ing Tunneling Microscopy	385			
		4.10.1	Principle of Operation	385			
		4.10.2	Basic Construction of an STM	387			
		4.10.3	Specific Features of Scanning Tunneling Microscopy	388			
		4.10.4	Atomic Force Microscope	389			
		4.10.5	Several Examples of Surface Images	390			
	4.11	Neutr	on Diffraction. Mössbauer Diffraction, and Scattering				
		of Nu	clear Particles in Crystals	392			
		4.11.1	Principles and Techniques of the Neutron				
			Diffraction Method	392			
		4.11.2	Investigation of the Atomic Structure	394			
		4.11.3	Investigation of the Magnetic Structure	397			
		4.11.4	Other Possibilities Offered by the Neutron-				
			Diffraction Method	400			
		4.11.5	Diffraction of Mössbauer Radiation	401			
		4.11.6	Particle Channeling and the Shadow Effect	403			
5.			Developments in Crystallography and its Methods				
	5.1	-	crystals	405			
		5.1.1	The Discovery of Quasicrystals	405			
		5.1.2	The Non-Traditional Symmetry of Aperiodic Objects	407			
		5.1.3	The One-Dimensional Quasicrystal $(d = 1, N = 2)$	409			
		5.1.4	Fourier Transform of Quasiperiodic Sequences	410			
		5.1.5	Two-Dimensional Quasicrystals and Their				
			Symmetry $(d = 2, N = 3, 4, 5,)$	412			
		5.1.6	Three-Dimensional Quasicrystals ($d = 3, N = 4, 5, 6,$)	41:			
		5.1.7	Structure Analysis of Quasicrystals	419			
		5.1.8	Order and Disorder in the Structure of Quasicrystals	424			
	5.2		nmensurately Modulated Structures	42			
	5.3		opment of Experimental Technique for X-Ray				
			ure Analysis	42			
		5.3.1	Powerful Sources of X-Ray Radiation	42			
		5.3.2	The Synchrotron Radiation Sources	429			
		5.3.3	Characteristics of Synchrotron Radiation	429			
		5.3.4	The Primary SR Beam	43			
		5.3.5	Laue Method for Crystallographic Data Acquisition				
			from Single Crystals	43			
		5.3.6	Some Results Obtained with Synchrotron				
			Radiation Sources	432			
		5.3.7	Diffractometers with Two-Dimensional				
			Position-Sensitive Detectors	43.			
	5.4		Studies of Crystal Surface	438			
		541	Surface Diffraction	440			

Contents	XXI
5.4.2 The Method of X-Ray Standing Waves	443
5.5 Methods for Analysis of Powder Diffraction Patterns	446
5.6 EXAFS Spectroscopy	451
5.6.1 The Fundamentals of the EXAFS	
Theory	451
5.6.2 Structure Information Extracted from EXAFS	
Spectra	452
5.6.3 Experimental Methods and Apparatus	454
References	455
Bibliography	466
Subject Index	473