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Control Theory in Physics and other Fields of Science

Concepts, Tools, and Applications

With 46 Figures



Springer

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