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Normally Hyperbolic Invariant Manifolds in Dynamical Systems

With 22 Illustrations

With the Assistance of
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Springer-Verlag

New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

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