# Contents

Preface — VII

Dependencies between the chapters ----- XVII

### Detailed plan —— XIX

# Acknowledgments and short history of this book ----- XXV

#### Notation conventions — XXVII

1	Basic transformations of categories: Hierarchy of <i>n</i> -dimensional levels — 1
1.1	The <i>n</i> -dimensional levels: A nonset-based definition of natural numbers —— <b>1</b>
1.2	Comma lifting for <i>n</i> -dimensional levels — 6
1.3	Introduction to comma-induction in the <i>n</i> -dimensional hierarchy — <b>16</b>
1.4	Comma-induction of the adjunctions —— 20
2	Comma-propagation transformations: Global categorial symmetries — 29
2.1	Introduction to general comma-propagation — 29
2.2	Comma-propagation of functors and natural transformations — 34
2.3	Comma-propagation of (co)limits — 40
2.4	Example: Comma-propagation and infinite hierarchy of small-complete categories —— <b>50</b>
2.5	An analogy between physical and abstract categorial Global symmetries: Adjunctions-as-fields — <b>57</b>
2.5.1	Analogy between adjunctions and metric tensor field with Einstein–Hilbert action — 60
2.5.2	Comma-propagation transformation symmetries —— 65
3	Arrows-to-objects conceptual transformation: Internal categorial
	symmetry 71
3.1	Introduction to categorial internal symmetry of primitive categorial concepts — <b>71</b>
3.2	Internal symmetry and metacategory 77
3.3	Conceptually closed categories: A topology — 83
3.4	Symmetry-extended categories <b> 94</b>
3.5	Symmetry hierarchy upper bound: Imploded categories — <b>101</b>
4	Internal symmetry and logical deduction 111
4.1	Natural deduction system — 111
4.1.1	First solution for tagging techniques of natural deduction — 111



- 4.1.2 Sequent-based solution for tagging techniques 116
- 4.2 Definition of symmetry-extended category ND for natural deduction 119
- 4.2.1 The properties of the covariant implication functor 125
- 4.2.2 Local Cartesian Closed Adjunction (pCCC) 128
- 4.2.3 The *n*-dimensional levels of natural deduction categories **133**
- 4.3 Symmetry-extended category **IC** for propositional intuitionistic calculus **138**

## 5 Internal symmetry and lambda calculus — 144

- 5.1 Introduction to lambda calculus 144
- 5.2 Reflexive objects in the Cartesian closed categories 149
- 5.3 Conceptually-closed CCC and its subcategories of idempotents 156
- 5.4 Internal categorial symmetry and fixed-point operators **166**
- 5.5 Topological K-theory, idempotent completion and internal categorial symmetry 169

# 6 Internal symmetry and theory of processes: Strong bisimulation of computation trees — 173

- 6.1 Introduction to transition systems and their bisimulations 173
- 6.2 Regular languages, automata and internal categorial symmetry 177
- 6.3 Symmetry-extended category of finite labeled trees 183
- 6.4 Internal symmetry of the category of relations **191**
- 6.5 Transition systems as fixed points in the *n*-dimensional level **Rel**<sub>3</sub> **196**
- 6.6 Strong bisimulations as fixed points in the *n*-dimensional level **Rel**<sub>3</sub> **201**
- 6.6.1 Mathematics via symmetry: Reduction of many valued into 2-valued logic 207
- 6.6.2 Many-valued knowledge invariance through modal logic transformations: Semantic reflection — 212

# 7 Internal symmetry and data integration theory — 220

- 7.1 DB (Database) category 220
- 7.1.1 Morphism properties of **DB** category 235
- 7.1.2 Power-view endofunctor and monad T 249
- 7.1.3 Duality 254
- 7.2 Objects of **DB**: Basic operations and equivalence relations **257**
- 7.2.1 Data federation operator in **DB** 257
- 7.2.2 Data separation operator in **DB** 258
- 7.2.3 The (strong) behavioral equivalence for databases 261
- 7.2.4 Weak observational equivalence for databases 263
- 7.3 Internal categorial symmetry of **DB** category **266**
- 7.3.1 (Co)products 270
- 7.3.2 (Co)Limits and exponentiation 274

- 7.3.3 Kleisli semantics for database mappings 282
- 7.4 Partial ordering for databases 288
- 7.4.1 Matching tensor product 292
- 7.4.2 Merging operator ---- 295
- 7.4.3 Universal algebra considerations ---- 298
- 7.4.4 Algebraic database lattice **302**
- 7.5 Enrichment 313
- 7.5.1 **DB** is a V-category enriched over itself **315**
- 7.5.2 Internalized Yoneda embedding 320

#### A Appendix — 323

- A.1 Introduction to lattices, algebras and logics 323
- A.1.1 Introduction to deductive logic and binary sequent calculus 327
- A.1.2 Introduction to first-order logic and Tarski's interpretations **329**
- A.1.3 Introduction to multimodal logics and Kripke semantics 333
- A.2 Basic category theory **334**
- A.3 Introduction to RDB, database mappings and **DB** category ----- **348**
- A.3.1 Basic database concepts 351
- A.3.2 Database observations: Idempotent power-view operator 357
- A.3.3 Logic versus algebras: Categorification by operads 360
- A.3.4 Sketches and functors into the **DB** category **363**
- A.3.5 Semantics of DB schema mappings: Information fluxes 370
- A.4 Introduction to field theory and symmetries 381
- A.4.1 Vector fields on curved differentiable manifolds 388
- A.4.2 Transformation of coordinates **392**

#### Bibliography — 395

Index — 403