

# Contents

## **1 Random Signals — 1**

- 1.1 Signal Classifications — 1
- 1.2 Correlation Function, Covariance Function, and Power Spectral Density — 6
  - 1.2.1 Autocorrelation Function, Autocovariance Function, and Power Spectral Density — 6
  - 1.2.2 Cross Correlation Function, Cross Covariance Function, and Cross Power Spectral Density — 10
- 1.3 Comparison and Discrimination between Two Random Signals — 13
  - 1.3.1 Independence, Uncorrelatedness, and Orthogonality — 14
  - 1.3.2 Gram-Schmidt Orthogonalization Process of Polynomial Sequence — 18
- 1.4 Linear System with Random Input — 19
  - 1.4.1 The Power Spectral Density of System Output — 19
  - 1.4.2 Narrow Band Bandpass Filter — 22
- Summary — 25
- Exercises — 25

## **2 Parameter Estimation Theory — 30**

- 2.1 Performance of Estimators — 30
  - 2.1.1 Unbiased and Asymptotic Unbiased Estimation — 31
  - 2.1.2 Effectiveness of Estimators — 33
- 2.2 Fisher Information and Cramér-Rao Inequality — 35
  - 2.2.1 Fisher Information — 35
  - 2.2.2 Cramér-Rao Lower Bound — 36
- 2.3 Bayes Estimation — 38
  - 2.3.1 Definition of Risk Function — 39
  - 2.3.2 Bayes Estimation — 40
- 2.4 Maximum Likelihood Estimation — 43
- 2.5 Linear Mean Squares Estimation — 47
- 2.6 Least Squares Estimation — 49
  - 2.6.1 Least Squares Estimation and Its Performance — 49
  - 2.6.2 Weighted Least Squares Estimation — 51
- Summary — 53
- Exercises — 53

## **3 Signal Detection — 57**

- 3.1 Statistical Hypothesis Testing — 57
  - 3.1.1 Basic Concepts of Signal Detection — 57
  - 3.1.2 Signal Detection Measures — 61

|          |                                                                            |
|----------|----------------------------------------------------------------------------|
| 3.1.3    | Decision Space — 65                                                        |
| 3.2      | Probability Density Function and Error Function — 68                       |
| 3.2.1    | Probability Density Function — 69                                          |
| 3.2.2    | Error Function and Complementary Error Function — 71                       |
| 3.3      | Probabilities of Detection and Error — 73                                  |
| 3.3.1    | Definitions of Detection and Error Probabilities — 74                      |
| 3.3.2    | Power Function — 77                                                        |
| 3.4      | Neyman-Pearson Criterion — 78                                              |
| 3.4.1    | Probabilities of False Alarm and Miss alarm in Radar Signal Detection — 79 |
| 3.4.2    | Neyman-Pearson Lemma and Neyman-Pearson Criterion — 82                     |
| 3.5      | Uniformly Most Power Criterion — 86                                        |
| 3.5.1    | Communication Signal Detection Problem — 86                                |
| 3.5.2    | Uniformly Most Power Test — 88                                             |
| 3.5.3    | Physical Meaning of UMP Criterion — 91                                     |
| 3.6      | Bayes Criterion — 92                                                       |
| 3.6.1    | Bayes Decision Criterion — 92                                              |
| 3.6.2    | Detection of Binary Signal Waveform — 95                                   |
| 3.6.3    | Detection Probability Analysis — 98                                        |
| 3.7      | Bayes Derived Criteria — 100                                               |
| 3.7.1    | Minimum Error Probability Criterion — 100                                  |
| 3.7.2    | Maximum A Posteriori Probability Criterion — 102                           |
| 3.7.3    | Minimax Criterion — 104                                                    |
| 3.8      | Multivariate Hypotheses Testing — 107                                      |
| 3.8.1    | Multivariate Hypotheses Testing Problem — 108                              |
| 3.8.2    | Bayes Criteria for Multiple Hypotheses Testing — 109                       |
| 3.9      | Multiple Hypothesis Testing — 110                                          |
| 3.9.1    | Error Rate of Multiple Hypothesis Testing — 111                            |
| 3.9.2    | Error Control Method of Multiple Hypothesis Testing — 114                  |
| 3.9.3    | Multiple Linear Regression — 116                                           |
| 3.9.4    | Multivariate Statistical Analysis — 120                                    |
|          | Summary — 125                                                              |
|          | Exercises — 125                                                            |
| <b>4</b> | <b>Modern Spectral Estimation — 132</b>                                    |
| 4.1      | Nonparametric Spectral Estimation — 132                                    |
| 4.1.1    | Discrete Stochastic Process — 133                                          |
| 4.1.2    | Non-parametric Power Spectrum Estimation — 134                             |
| 4.2      | Stationary ARMA Process — 135                                              |
| 4.3      | Power Spectral Density of Stationary Process — 141                         |
| 4.3.1    | Power Spectral Density of ARMA Process — 141                               |
| 4.3.2    | Power Spectrum Equivalence — 146                                           |

|       |                                                                       |     |
|-------|-----------------------------------------------------------------------|-----|
| 4.4   | ARMA Spectrum Estimation —                                            | 149 |
| 4.4.1 | Two Linear Methods for ARMA Power Spectrum Estimation —               | 150 |
| 4.4.2 | Modified Yule-Walker Equation —                                       | 151 |
| 4.4.3 | Singular Value Decomposition Method for AR Order Determination —      | 154 |
| 4.4.4 | Total Least Squares Method for AR Parameter Estimation —              | 157 |
| 4.5   | ARMA Model Identification —                                           | 160 |
| 4.5.1 | MA Order Determination —                                              | 160 |
| 4.5.2 | MA Parameter Estimation —                                             | 163 |
| 4.6   | Maximum Entropy Spectrum Estimation —                                 | 165 |
| 4.6.1 | Burg Maximum Entropy Spectrum Estimation —                            | 165 |
| 4.6.2 | Levinson Recursion —                                                  | 168 |
| 4.6.3 | Burg Algorithm —                                                      | 173 |
| 4.6.4 | Burg Maximum Entropy Spectrum Analysis and ARMA Spectrum Estimation — | 174 |
| 4.7   | Pisarenko Harmonic Decomposition Method —                             | 177 |
| 4.7.1 | Pisarenko Harmonic Decomposition —                                    | 177 |
| 4.7.2 | ARMA Modeling Method for Harmonic Recovery —                          | 180 |
| 4.8   | Extended Prony Method —                                               | 182 |
|       | Summary —                                                             | 188 |
|       | Exercises —                                                           | 188 |

|          |                                                     |            |
|----------|-----------------------------------------------------|------------|
| <b>5</b> | <b>Adaptive Filter —</b>                            | <b>193</b> |
| 5.1      | Matched Filter —                                    | 193        |
| 5.1.1    | Matched Filter —                                    | 194        |
| 5.1.2    | Properties of Matched Filter —                      | 199        |
| 5.1.3    | Implementation of Matched Filter —                  | 200        |
| 5.2      | Continuous Time Wiener Filter —                     | 201        |
| 5.3      | Optimal Filtering Theory and Wiener Filter —        | 203        |
| 5.3.1    | Linear Optimal Filter —                             | 203        |
| 5.3.2    | Orthogonality Principle —                           | 205        |
| 5.3.3    | Wiener Filter —                                     | 206        |
| 5.4      | Kalman Filter —                                     | 209        |
| 5.4.1    | Kalman Filtering Problem —                          | 209        |
| 5.4.2    | Innovation Process —                                | 210        |
| 5.4.3    | Kalman Filtering Algorithm —                        | 212        |
| 5.5      | LMS Adaptive Algorithms —                           | 214        |
| 5.5.1    | Descent Algorithm —                                 | 214        |
| 5.5.2    | LMS Algorithm and Its Basic Variants —              | 216        |
| 5.5.3    | Decorrelation LMS Algorithm —                       | 217        |
| 5.5.4    | Selection of the Learning Rate Parameter —          | 221        |
| 5.5.5    | Statistical Performance Analysis of LMS Algorithm — | 223        |

|          |                                                                   |            |
|----------|-------------------------------------------------------------------|------------|
| 5.5.6    | Tracking Performance of LMS Algorithm —                           | 225        |
| 5.6      | RLS Adaptive Algorithm —                                          | 228        |
| 5.6.1    | RLS Algorithm —                                                   | 229        |
| 5.6.2    | Comparison between RLS Algorithm and Kalman Filtering Algorithm — | 232        |
| 5.6.3    | Statistical Performance Analysis of RLS Algorithm —               | 234        |
| 5.6.4    | Fast RLS Algorithm —                                              | 235        |
| 5.7      | Adaptive Line Enhancer and Notch Filter —                         | 237        |
| 5.7.1    | Transfer Functions of Line Enhancer and Notch Filter —            | 237        |
| 5.7.2    | Adaptive Notch Filter based on Lattice IIR Filter —               | 239        |
| 5.8      | Generalized Sidelobe Canceller —                                  | 242        |
| 5.9      | Blind Adaptive Multiuser Detection —                              | 244        |
| 5.9.1    | Canonical Representation of Blind Multiuser Detection —           | 245        |
| 5.9.2    | LMS and RLS Algorithms for Blind Multiuser Detection —            | 246        |
| 5.9.3    | Kalman Adaptive Algorithm for Blind Multiuser Detection —         | 249        |
|          | Summary —                                                         | 253        |
|          | Exercises —                                                       | 253        |
| <b>6</b> | <b>Higher-Order Statistical Analysis —</b>                        | <b>259</b> |
| 6.1      | Moments and Cumulants —                                           | 259        |
| 6.1.1    | Definition of Higher-order Moments and Cumulants —                | 259        |
| 6.1.2    | Higher-order Moments and Cumulants of Gaussian Signal —           | 262        |
| 6.1.3    | Transformation Relationships between Moments and Cumulants —      | 263        |
| 6.2      | Properties of Moments and Cumulants —                             | 265        |
| 6.3      | Higher-order Spectra —                                            | 270        |
| 6.3.1    | Higher-order Moment Spectra and Higher-order Cumulant Spectra —   | 270        |
| 6.3.2    | Bispectrum Estimation —                                           | 272        |
| 6.4      | Non-Gaussian Signal and Linear System —                           | 276        |
| 6.4.1    | Sub-Gaussian and Super-Gaussian Signal —                          | 276        |
| 6.4.2    | Non-Gaussian Signal Passing Through Linear System —               | 277        |
| 6.5      | FIR System Identification —                                       | 280        |
| 6.5.1    | RC Algorithm —                                                    | 280        |
| 6.5.2    | Cumulant Algorithm —                                              | 284        |
| 6.5.3    | MA Order Determination —                                          | 288        |
| 6.6      | Identification of Causal ARMA Models —                            | 289        |
| 6.6.1    | Identification of AR Parameters —                                 | 290        |
| 6.6.2    | MA order Determination —                                          | 293        |
| 6.6.3    | Estimation of MA Parameters —                                     | 296        |
| 6.7      | Harmonic Retrieval in Colored Noise —                             | 299        |
| 6.7.1    | Cumulant Definition for Complex Signal —                          | 300        |
| 6.7.2    | Cumulants of Harmonic Process —                                   | 302        |

|          |                                                             |            |
|----------|-------------------------------------------------------------|------------|
| 6.7.3    | Harmonic Retrieval in Colored Gaussian Noise —              | 304        |
| 6.7.4    | Harmonic Retrieval in Colored Non-Gaussian Noise —          | 305        |
| 6.8      | The Adaptive Filtering of Non-Gaussian Signal —             | 311        |
| 6.9      | Time Delay Estimation —                                     | 312        |
| 6.9.1    | The Generalized Correlation Method —                        | 312        |
| 6.9.2    | Higher-Order Statistics Method —                            | 314        |
| 6.10     | Application of Bispectrum in Signal Classification —        | 319        |
| 6.10.1   | The Integrated Bispectra —                                  | 319        |
| 6.10.2   | Selected Bispectra —                                        | 322        |
|          | Summary —                                                   | 325        |
|          | Exercises —                                                 | 325        |
| <b>7</b> | <b>Linear Time-Frequency Transform —</b>                    | <b>329</b> |
| 7.1      | Local Transformation of Signals —                           | 329        |
| 7.2      | Analytic Signal and Instantaneous Physical Quantity —       | 332        |
| 7.2.1    | Analytic Signal —                                           | 333        |
| 7.2.2    | Baseband Signal —                                           | 335        |
| 7.2.3    | Instantaneous Frequency and Group Delay —                   | 337        |
| 7.2.4    | Exclusion Principle —                                       | 339        |
| 7.3      | Short-Time Fourier Transform —                              | 342        |
| 7.3.1    | The Continuous Short-Time Fourier Transform —               | 342        |
| 7.3.2    | The Discrete Short-Time Fourier Transform —                 | 345        |
| 7.4      | Gabor Transform —                                           | 348        |
| 7.4.1    | The Continuous Gabor Transform —                            | 348        |
| 7.4.2    | The Discrete Gabor Transform —                              | 354        |
| 7.5      | Fractional Fourier transform —                              | 357        |
| 7.5.1    | Definition and Properties of Fractional Fourier Transform — | 358        |
| 7.5.2    | Calculation of Fractional Fourier Transform —               | 361        |
| 7.6      | Wavelet Transform —                                         | 362        |
| 7.6.1    | Physical Considerations of Wavelets —                       | 362        |
| 7.6.2    | The Continuous Wavelet Transform —                          | 364        |
| 7.6.3    | Discretization of Continuous Wavelet Transform —            | 366        |
| 7.7      | Wavelet Analysis and Frame Theory —                         | 367        |
| 7.7.1    | Wavelet Analysis —                                          | 368        |
| 7.7.2    | Frame Theory —                                              | 372        |
| 7.8      | Multiresolution Analysis —                                  | 377        |
| 7.9      | Orthogonal Filter Banks —                                   | 381        |
| 7.9.1    | Orthogonal wavelet —                                        | 381        |
| 7.9.2    | Fast Orthogonal Wavelet Transform —                         | 384        |
| 7.10     | Biorthogonal Filter Bank —                                  | 388        |
| 7.10.1   | Biorthogonal Multiresolution Analysis —                     | 388        |
| 7.10.2   | Design of Biorthogonal Filter Banks —                       | 391        |

|        |                                                      |     |
|--------|------------------------------------------------------|-----|
| 7.10.3 | Biorthogonal Wavelet and Fast Biorthogonal Transform | 393 |
|        | Summary                                              | 397 |
|        | Exercises                                            | 397 |

**8 Quadratic Time-frequency Distribution — 401**

|       |                                                                       |     |
|-------|-----------------------------------------------------------------------|-----|
| 8.1   | The General Theory of Time-frequency Distribution                     | 401 |
| 8.1.1 | Definition of the Time-frequency Distribution                         | 402 |
| 8.1.2 | Basic Properties of Time-frequency Distribution                       | 403 |
| 8.2   | The Wigner-Ville Distribution                                         | 405 |
| 8.2.1 | Mathematical Properties                                               | 406 |
| 8.2.2 | Relationship to Evolutive Spectrum                                    | 408 |
| 8.2.3 | Signal Reconstruction based on Wigner-Ville Distribution              | 410 |
| 8.3   | Ambiguity Function                                                    | 412 |
| 8.4   | Cohen's Class Time-frequency Distribution                             | 416 |
| 8.4.1 | Definition of Cohen's Class Time-frequency Distribution               | 416 |
| 8.4.2 | Requirements for Kernel Function                                      | 420 |
| 8.5   | Performance Evaluation and Improvement of Time-frequency Distribution | 422 |
| 8.5.1 | Time-frequency Aggregation                                            | 422 |
| 8.5.2 | Cross-Term Suppression                                                | 424 |
| 8.5.3 | Other Typical Time-frequency Distributions                            | 428 |
|       | Summary                                                               | 435 |
|       | Exercises                                                             | 436 |

**9 Blind Signal Separation — 438**

|       |                                                                   |     |
|-------|-------------------------------------------------------------------|-----|
| 9.1   | Basic Theory of Blind Signal Processing                           | 438 |
| 9.1.1 | A Brief Introduction to Blind Signal Processing                   | 438 |
| 9.1.2 | Model and Basic Problem of BSS                                    | 439 |
| 9.1.3 | Basic Assumption and Performance Requirement of BSS               | 442 |
| 9.2   | Adaptive Blind Signal Separation                                  | 443 |
| 9.2.1 | Neural Network Implementation of Adaptive Blind Signal Separation | 444 |
| 9.2.2 | Quasiidentity Matrix and Contrast Function                        | 447 |
| 9.3   | Independent Component Analysis                                    | 448 |
| 9.3.1 | Mutual Information and Negentropy                                 | 448 |
| 9.3.2 | Natural Gradient Algorithm                                        | 451 |
| 9.3.3 | Implementation of the Natural Gradient Algorithm                  | 454 |
| 9.3.4 | Fixed-Point Algorithm                                             | 458 |
| 9.4   | Nonlinear Principal Component Analysis                            | 460 |
| 9.4.1 | Pre-whitening                                                     | 460 |
| 9.4.2 | Linear Principal Component Analysis                               | 461 |
| 9.4.3 | Nonlinear Principal Component Analysis                            | 464 |

|           |                                                                                |            |
|-----------|--------------------------------------------------------------------------------|------------|
| 9.5       | Joint Diagonalization of Matrices —                                            | 465        |
| 9.5.1     | Blind Signal Separation and Joint Diagonalization of Matrices —                | 466        |
| 9.5.2     | Orthogonal Approximate Joint Diagonalization —                                 | 468        |
| 9.5.3     | Nonorthogonal Approximate Joint Diagonalization —                              | 470        |
| 9.6       | Blind Signal Extraction —                                                      | 472        |
| 9.6.1     | Orthogonal Blind Signal Extraction —                                           | 472        |
| 9.6.2     | Nonorthogonal Blind Signal Extraction —                                        | 473        |
| 9.7       | Blind Signal Separation of Convolutively Mixed Sources —                       | 476        |
| 9.7.1     | Convolutively Mixed Sources —                                                  | 477        |
| 9.7.2     | Time Domain Blind Signal Separation of Convolutively Mixed Sources —           | 479        |
| 9.7.3     | Frequency Domain Blind Signal Separation of Convolutively Mixed Sources —      | 482        |
| 9.7.4     | Time-Frequency Domain Blind Signal Separation of Convolutively Mixed Sources — | 484        |
|           | Summary —                                                                      | 491        |
|           | Exercises —                                                                    | 492        |
| <b>10</b> | <b>Array Signal Processing —</b>                                               | <b>497</b> |
| 10.1      | Coordinate Representation of Array —                                           | 497        |
| 10.1.1    | Array and Noise —                                                              | 498        |
| 10.1.2    | Coordinate System of Array —                                                   | 499        |
| 10.2      | Beamforming and Spatial Filtering —                                            | 501        |
| 10.2.1    | Spatial FIR Filter —                                                           | 502        |
| 10.2.2    | Broadband Beamformer —                                                         | 505        |
| 10.2.3    | Analogy and Interchange between Spatial FIR Filter and Beamformer —            | 508        |
| 10.3      | Linearly-Constrained Adaptive Beamformer —                                     | 512        |
| 10.3.1    | Classical Beamforming —                                                        | 512        |
| 10.3.2    | Direct Implementation of Adaptive Beamforming —                                | 516        |
| 10.3.3    | Generalized Sidelobe Canceling Form of Adaptive Beamforming —                  | 518        |
| 10.4      | Multiple Signal Classification (MUSIC) —                                       | 522        |
| 10.4.1    | Spatial Spectrum —                                                             | 522        |
| 10.4.2    | Signal Subspace and Noise Subspace —                                           | 524        |
| 10.4.3    | MUSIC Algorithm —                                                              | 526        |
| 10.5      | Extensions of MUSIC Algorithm —                                                | 529        |
| 10.5.1    | Decoherent MUSIC Algorithm —                                                   | 529        |
| 10.5.2    | Root-MUSIC Algorithm —                                                         | 532        |
| 10.5.3    | Minimum Norm Algorithm —                                                       | 533        |
| 10.5.4    | First Principal Vector MUSIC Algorithm —                                       | 535        |
| 10.6      | Beamspace MUSIC Algorithm —                                                    | 537        |
| 10.6.1    | BS-MUSIC Algorithm —                                                           | 537        |

|        |                                                                     |            |
|--------|---------------------------------------------------------------------|------------|
| 10.6.2 | Comparison of BS-MUSIC and ES-MUSIC —                               | <b>540</b> |
| 10.7   | Estimating Signal Parameters via Rotational Invariance Techniques — | <b>543</b> |
| 10.7.1 | Basic ESPRIT Algorithm —                                            | <b>544</b> |
| 10.7.2 | Element Space ESPRIT —                                              | <b>549</b> |
| 10.7.3 | TLS-ESPRIT —                                                        | <b>552</b> |
| 10.7.4 | Beamspace ESPRIT Method —                                           | <b>553</b> |
| 10.8   | Unitary ESPRIT and Its Extensions —                                 | <b>556</b> |
| 10.8.1 | Unitary ESPRIT Algorithm —                                          | <b>556</b> |
| 10.8.2 | Beamspace Unitary ESPRIT Algorithm —                                | <b>560</b> |
|        | Summary —                                                           | <b>562</b> |
|        | Exercises —                                                         | <b>562</b> |
| <br>   |                                                                     |            |
|        | <b>Index —</b>                                                      | <b>565</b> |
| <br>   |                                                                     |            |
|        | <b>Bibliography —</b>                                               | <b>577</b> |