

Contents

Part I Introduction

Introduction to Superfluid Vortices and Turbulence

<i>C.F. Barenghi</i>	3
1 The Two-Fluid Model	3
2 Quantized Vortex Lines	4
3 Modelling the Vortex Lines	5
3.1 Microscopic Model	5
3.2 Mesoscopic Model	6
3.3 Macroscopic Model	8
4 Turbulence	9
4.1 Turbulent Counterflows	9
4.2 Turbulent Coflows	9
5 Motion of Superfluid Vortices for a Given Normal Fluid	10
6 Motion of the Normal Fluid at Given Superfluid Vortices	11
7 Fully Coupled Motion of Superfluid Vortices and Normal Fluid	12
8 Discussion	13
References	13

Part II Turbulence Experiments

An Introduction to Experiments on Superfluid Turbulence

<i>R.J. Donnelly</i>	17
1 Introduction	17
2 Update on Pipe Flow	17
3 Update on Towed Grid Experiments	20
3.1 The Nature of Grid Turbulence in Helium II	20
3.2 Four Regimes of Decaying Grid Turbulence in Helium II	23
4 Agenda for the Future	24
4.1 The University of Oregon 6 cm Wind Tunnel	24
4.2 Wind Tunnels for Model Testing	24
4.3 Tow Tanks	25
5 Challenges for the Future	26
5.1 The Challenge of Instrumentation	26
5.2 Challenges for Understanding Counterflow Turbulence	29

5.3	Challenges for Understanding Periodic Boundary Layer Experiments	31
5.4	Instrumentation to Detect Vortices Below 1 K	33
5.5	The Normal Fluid and the Vortex Tangle	33
5.6	Flow over Blunt Objects, Testing Models such as Submarines . . .	34
	References	34

The Experimental Evidence for Vortex Nucleation in ^4He

	<i>É. Varoquaux, O. Avenel, Y. Mukharsky, P. Hakonen</i>	36
1	Single Vortex Nucleation	36
2	Multiple Slips and Collapses	44
	References	48

Applications of Superfluid Helium in Large-Scale Superconducting Systems

	<i>S.W. Van Sciver</i>	51
1	Introduction	51
2	Superconducting Systems That Use He II Cooling	53
2.1	Accelerator Magnet System for LHC	53
2.2	High Field Solenoid for the NHMFL 45-T Hybrid	54
2.3	RF Cavity Systems for the TESLA Electron Collider	55
3	Application Relevant He II Properties	57
3.1	Second Sound Pulse Transport	57
3.2	Transient and Steady Transport in the Mutual Friction Regime . .	58
3.3	The He II Energy Equation	59
3.4	Fluid Dynamics of Forced Flow He II	60
3.5	He II/Vapor Two Phase Flow	62
3.6	Fountain Effect (Fluid Management)	62
4	Conclusions	63
	References	64

The Temperature Dependent Drag Crisis on a Sphere in Flowing Helium II

	<i>Y.S. Choi, M.R. Smith, S.W. Van Sciver</i>	66
1	Introduction	66
2	Experimental Apparatus and Protocol	69
3	Results and Discussion	69
4	Conclusion	71
	References	72

Experiments on Quantized Turbulence at mK Temperatures

	<i>S.I. Davis, P.C. Hendry, P.V.E. McClintock, H. Nichol</i>	73
1	Background	73
2	Creation and Detection of Vortices	74
3	The Experiment	75
4	Preliminary Results	76
5	Discussion	76

6	Conclusions	78
	References	79

Grid-Generated He II Turbulence in a Finite Channel – Experiment

	<i>J.J. Niemela, L. Skrbek, S.R. Stalp</i>	80
1	Introduction	80
2	Experimental Setup	80
	References	86

Intermittent Switching Between Turbulent and Potential Flow Around a Sphere in He II at mK Temperatures

	<i>M. Niemetz, H. Kerscher, W. Schoepe</i>	87
1	Experiment	87
2	Results	88
	2.1 Stable Turbulent Flow	88
	2.2 Intermittent Switching	89
	2.3 Turbulent Phases	90
	2.4 Laminar Phases	91
3	Conclusion	93
	References	94

Part III Vortex Dynamics

Vortex Filament Methods for Superfluids

	<i>D.C. Samuels</i>	97
1	Introduction	97
2	Vortex Filament Motion	98
	2.1 The Biot–Savart Law and the Local Induction Approximation ...	99
	2.2 Boundary Conditions	102
	2.3 Meshing of the Filaments	103
3	Reconnections of Filaments	105
4	Analysis of the Superfluid Flow	107
5	Alternative Approaches	110
6	Conclusions: What Needs to Be Done	111
	References	112

Introduction to HVBK Dynamics

	<i>D.D. Holm</i>	114
1	HVBK Equations	114
2	Incompressible Renormalized HVBK Flows	120
3	Rotating Frame Renormalized HVBK Equations	123
	Appendix: Lie-Poisson Hamiltonian Formulation	125
	References	130

**Magnus Force, Aharonov–Bohm Effect,
and Berry Phase in Superfluids**

<i>E. Sonin</i>	131
1 Introduction	131
2 Gross–Pitaevskii Theory and Two-Fluid Hydrodynamics	132
3 Interaction of Phonons with a Vortex in Hydrodynamics	133
4 Momentum Balance in the Two-Fluid Hydrodynamics	135
5 Magnus Force and the Berry Phase	136
References	137

**Using the HVBK Model
to Investigate the Couette Flow of Helium II**

<i>K.L. Henderson</i>	138
1 Introduction	138
2 Linear Theory	140
3 Nonlinear Solutions	141
3.1 Infinite Cylinder Assumption	141
3.2 Unit Aspect Ratio	143
4 Discussion	145
References	145

Part IV Turbulence Theory

An Introduction to the Theory of Superfluid Turbulence

<i>W.F. Vinen</i>	149
1 Introduction	149
2 Counterflow Turbulence	150
3 Grid Turbulence in Superfluid Helium	152
3.1 Measurements of the Decay of Vortex Lines, and the Quasi-classical Model	152
3.2 Superfluid Turbulence on Length Scales Larger than the Vortex Line Spacing	153
3.3 The Turbulent Energy Spectra in Superfluid Grid Turbulence ...	154
3.4 Superfluid Turbulence at Very Low Temperatures	155
3.5 Dissipation at Higher Temperatures	158
4 Summary and Conclusions	159

**Numerical Methods for Coupled Normal-Fluid
and Superfluid Flows in Helium II**

<i>O.C. Idowu, D. Kivotides, C.F. Barenghi, D.C. Samuels</i>	162
1 Introduction	162
2 The Self-Consistent Equation of Motion	162
3 Numerical Methods for 2-D Flows	163
3.1 The Normal-Fluid Flow in 2-D	165
3.2 Delta Function Forcing on a Grid	166

3.3	Extrapolation of the Normal-Fluid Flow in the Neighbourhood of the Superfluid Vortex Line	166
3.4	Numerical Stability and Time Stepping	168
4	Results in 2-D Flows	168
5	Numerical Methods for 3-D Flows	170
5.1	The Free Normal-Fluid	170
5.2	The Superfluid	172
5.3	The Interaction Modelling	174
5.4	Preliminary Results in 3-D Flows	174
6	Discussion and Conclusion	175
	References	176

From Vortex Reconnections to Quantum Turbulence

<i>T. Lipniacki</i>	177
1 Introduction	177
2 Vortex Motion Following Reconnection	178
2.1 The Case $v_{ns} = 0$	178
2.2 The Case $v_{ns} = \text{const} \neq 0$	179
3 The Model	181
4 Results	182
References	183

Vortices and Stability in Superfluid Boundary Layers

<i>S.P. Godfrey, D.C. Samuels, C.F. Barenghi</i>		184
1	Introduction	184
1.1	The Two-Fluid Model	184
2	Boundary Layer Vortices	184
2.1	Properties of the Vortex Line Solutions	186
2.2	Discussion	187
3	Stability Analysis	188
3.1	Linear Stability	188
3.2	Stability Results	189
3.3	Discussion	189
References		190

Grid Generated He II Turbulence in a Finite Channel – Theoretical Interpretation

<i>L. Skrbek, J.J. Niemela</i>	191
1 Introduction	191
2 The Spectral Decay Model	193
References	197

Vortex Tangle Dynamics Without Mutual Friction in Superfluid ^4He

<i>M. Tsubota, T. Araki, S.K. Nemirovskii</i>	198
1 Introduction	198
2 Vortex Wave Cascade Process	198

3	Cascade Process in the Vortex Tangle	200
3.1	Decay of the Vortex Tangle	200
3.2	Comparison with the Vinen's Equation	203
	References	204

Applications of the Gaussian Model of the Vortex Tangle in the Superfluid Turbulent He II

	<i>S.K. Nemirovskii, M.V. Nedoboiko</i>	205
1	Introduction	205
2	Constructing the Trial Distribution Function	206
3	Hydrodynamic Impulse of the Vortex Tangle	208
4	Energy of the Vortex Tangle	209
5	Conclusion	211
	References	211

Stochastic Dynamics of a Vortex Loop.

Thermal Equilibrium

	<i>S.K. Nemirovskii, L.P. Kondaurova, M. Tsubota</i>	212
1	Introduction and Scientific Background	212
2	Langevin Equation	213
3	Fokker–Planck Equation	216
4	Possible Violation of Thermal Equilibrium	217
	References	218

Stochastic Dynamics of Vortex Loop.

Large-Scale Stirring Force

	<i>S.K. Nemirovskii, A.Ja. Baltsevich</i>	219
1	Introduction	219
2	Analytical Investigation	220
3	Conservation Laws and Pair Correlators	221
4	Some Numerical Results	225
	References	225

Nonequilibrium Vortex Dynamics in Superfluid Phase Transitions and Superfluid Turbulence

	<i>H.-C. Chu, G.A. Williams</i>	226
1	Introduction	226
2	Quenched Superfluid Transition	227
3	Superfluid Turbulence	230
4	Three Dimensions	232
	References	232

Part V The NLSE and Superfluidity

The Nonlinear Schrödinger Equation as a Model of Superfluidity

<i>P.H. Roberts, N.G. Berloff</i>	235
1 Introduction	235
2 The Fluid Equations	237
3 Shortcomings of the GP Model	238
4 Vortices	241
5 Superfluid Turbulence; Vortex Line Reconnection	243
6 Intrinsic Vortex Nucleation	244
7 Capture of Impurities by Vortex Lines	249
8 Nonlocal Models	252
9 Conclusions	254
References	256

Vortex Nucleation and Limit Speed for a Flow Passing Nonlinearly Around a Disk in the Nonlinear Schrödinger Equation

<i>S. Rica</i>	258
1 Introduction and Formulation of the Problem	258
2 Critical Velocities	260
3 Flow Around a Disk via a Janzen–Rayleigh Expansion	262
4 Unstable Solutions	264
5 The Euler–Tricomi Equation near the Transonic Region	265
References	267

Vortices in Nonlocal Condensate Models of Superfluid Helium

<i>N.G. Berloff, P.H. Roberts</i>	268
1 Introduction	268
2 Applicability of the Generalized Gross–Pitaevskii Model	269
3 Nonlocal Nonlinear Schrödinger Equation	271
4 Vortex Nucleation and Roton Emission	273
5 Conclusions	274
References	275

Ginzburg–Landau Description of Vortex Nucleation in a Rotating Superfluid

<i>I. Aranson, V. Steinberg</i>	276
1 Introduction	276
2 Spin-Up and Nucleation of Vortices in Superfluid Helium	277
3 Stability of Multicharged Vortices	279
4 Nucleation of Vortices by Rapid Thermal Quench	281
References	282

Weak Turbulence Theory for the Gross–Pitaevskii Equation

<i>S. Nazarenko, Y. Lvov, R. West</i>	283
1 Motivation and Background	283
2 Weak Turbulence Theory for NLSE	284
3 Linear Dynamics of the GPE	285
3.1 Without a Condensate	286
3.2 With a Condensate	286
4 Applicability of WKB Descriptions	287
5 Weakly Nonlinear GPE Waves	289
References	289

Dissipative Vortex Dynamics and Magnus Force

<i>L.M. Pismen</i>	290
1 Basic Equations	290
2 Magnus Force	292
3 Three-Dimensional Effects	294
4 Failure of Mechanistic Reduction	295
References	296

Transition to Dissipation**in Two- and Three-Dimensional Superflows**

<i>C. Huepe, C. Nore, M.-E. Brachet</i>	297
1 Introduction	297
2 Definition of the System	298
3 Numerical Methods	299
4 Bifurcation Diagram and Scaling in 2D	299
5 Subcriticality and Vortex-Stretching in 3D	301
References	303

Part VI Bose–Einstein Condensation**Motion of Objects Through Dilute Bose–Einstein Condensates**

<i>C.S. Adams, B. Jackson, M. Leadbeater, J.F. McCann, T. Winiecki</i>	307
1 Introduction	307
2 Fluid Equations	308
3 Time-Independent Solutions in the Object Frame	309
4 The Critical Velocity	311
5 Vortex Shedding and Drag	313
6 The Critical Velocity in Inhomogeneous Condensates	314
7 Comparison to Ions in Helium	316
8 Conclusion	318
References	318

Stability of a Vortex in a Rotating Trapped Bose–Einstein Condensate

<i>A.L. Fetter, A.A. Svidzinsky</i>	320
1 Time-Dependent Gross–Pitaevskii Equation	320
1.1 Equivalent Hydrodynamics of Compressible Isentropic Fluid.....	320
1.2 Thomas–Fermi Limit for Large Condensates.....	321
2 Energy of a Vortex in a Large Rotating Trap	321
3 Small-Amplitude Excitation of a Vortex in a Rotating Trap	322
3.1 Stability of a Vortex	323
3.2 Splitting of Normal-Mode Frequencies Caused by a Vortex	323
4 Vortex Dynamics	324
4.1 Dynamics of Straight Vortex	324
4.2 Inclusion of Curvature.....	325
References	326

Kinetics of Strongly Non-equilibrium Bose–Einstein Condensation

<i>B. Svistunov</i>	327
1 Introduction	327
2 Kinetic Regime	329
3 Coherent Regime	330
4 External Potential	331
References	333

Quantum Nucleation of Phase Slips in Bose–Einstein Condensates

<i>H.P. Büchler, V.B. Geshkenbein, G. Blatter</i>	334
1 Introduction	334
2 Effective Action	336
3 Finite Size Effects	338
References	340

Part VII Vortex Reconnections and Classical Aspects

Vortex Reconnection in Normal and Superfluids

<i>J. Koplik</i>	345
1 Introduction	345
2 Some Vortex Generalities	345
3 The Importance of Reconnection	348
4 Reconnection in Normal Fluids.....	350
5 Reconnection in Superfluids	352
6 Conclusions.....	356
References	356

Helicity in MHD and Hydro Reconnection

<i>A. Brandenburg, R.M. Kerr</i>	358
1 Introduction	358
2 Dissipation of Energy and Helicity	359
3 Interlocked Flux Rings	360
4 Orthogonal Vortex Tubes	364
References	365

Tropicity and Complexity Measures for Vortex Tangles

<i>R.L. Ricca</i>	366
1 Vortex Structures and Tangles in Classical and Quantized Vortex Flows	366
2 Measures of Tropicity for Vortex Tangles: Tubeness, Sheetiness and Bulkiness	367
3 Measures of Geometric Complexity: Directional Alignment and Writhing	368
4 Algebraic Measure of Structural Complexity: Average Crossing Number	369
5 Measures of Topological Entanglement: Kinetic Helicity and Directional Linking	370
6 Relationships Between Complexity Measures and Energy Levels	371
References	372

The Geometry of Magnetic and Vortex Reconnection

<i>G. Hornig</i>	373
1 Introduction	373
2 Magnetic Reconnection	374
3 Vortex Reconnection	377
4 Conclusions	379
References	379

Current-Sheet Formation**near a Hyperbolic Magnetic Neutral Line**

<i>B.K. Shivamoggi</i>	381
1 Introduction	381
2 Current-Sheet Formation at a Hyperbolic Magnetic Neutral Line in a Stagnation-Point Plasma Flow	382
3 Effect of a Uniform Shear-Strain in the Plasma Flow	385
4 Discussion	387
References	387

Nonlocality in Turbulence

<i>A. Tsinober</i>	389
1 Introduction and Simple Examples	389
2 Different Aspects of Nonlocality	391
2.1 Direct Coupling Between Large and Small Scales	392
3 Concluding Remarks	395
References	395

Part VIII Helium 3 and Other Systems

Quantized Vorticity in Superfluid $^3\text{He-A}$:**Structure and Dynamics**

<i>R. Blaauwgeers, V.B. Eltsov, M. Krusius, J. Ruohio, R. Schanen</i>	399
1 Superfluid ^3He	399
2 Order-Parameter Texture and Superflow in $^3\text{He-A}$	401
3 Double-Quantum Vortex Line	403
4 Vortex Sheet	409
5 Dynamic Response	413
6 Summary and Future Work	419
References	419

Vortices in Metastable ^4He Films

<i>R. Blossey</i>	421
1 Wetting Properties of ^4He on Weak-Binding Alkali Metals	421
1.1 Wetting Transitions of Liquid Helium	421
1.2 Interface Model Description of Wetting Transitions	422
2 Lifetime of an Undercooled Film	423
3 Application to $^4\text{He}/\text{Cs}$	424
4 Conclusions	426
References	427

Quantum Hall Effect Breakdown Steps and Possible Analogies with Classical and Superfluid Hydrodynamics

<i>L. Eaves</i>	428
1 Introduction	428
2 Model and Comparison with Experiment	430
3 Analogies with Classical and Quantum Fluids	434
4 The Breakdown Steps and Their Relation to Other Types of QHE Breakdown	435
5 Summary	435
References	436

Atomic Bose Condensate with a Spin Structure: The Use of the Bloch State

<i>H. Kuratsuji</i>	438
1 Introduction	438
2 Order Parameter and Lagrangian	439
3 Hydrodynamical Equation	440
4 Vortex State	441
4.1 The Profile of a Single Vortex	441
4.2 Vortex Dynamics	442
5 Summary	444
References	444

Quantum Dynamics of Vortex–Antivortex Pairs in a Circular Box

V. Penna 445

1 Introduction 445

2 Canonical Quantization of Planar Vortices 446

 2.1 The Spectrum of Unbounded Vortex Pairs 447

3 Pair Quantum Dynamics in a Circular Box 448

 3.1 Spectral Structure of Low Energy States 449

References 451

Index of Topics 453