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An Introduction to Riemann Surfaces, Algebraic Curves and Moduli Spaces

Second Edition

With 21 Figures



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