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Topics and Methods in Condensed Matter Theory

From Basic Quantum Mechanics
to the Frontiers of Research

With 73 Figures and 4 Tables



Springer

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