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# Brownian Agents and Active Particles

Collective Dynamics  
in the Natural and Social Sciences

With a Foreword by J. Doyne Farmer

With 192 Figures



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