

Contents

Introduction	1
1 Preliminaries	3
1.1 Brief Sketch of Lebesgue's Integral	3
1.2 Convergence Concepts for Random Variables	7
1.3 The Lebesgue-Stieltjes Integral	10
1.4 Exercises	13
2 Introduction to Itô-Calculus	15
2.1 Stochastic Calculus vs. Classical Calculus	15
2.2 Quadratic Variation and 1-dimensional Itô-Formula	18
2.3 Covariation and Multidimensional Itô-Formula	26
2.4 Examples	31
2.5 First Application to Financial Markets	33
2.6 Stopping Times and Local Martingales	36
2.7 Local Martingales and Semimartingales	44
2.8 Itô's Representation Theorem	49
2.9 Application to Option Pricing	50
3 The Girsanov Transformation	55
3.1 Heuristic Introduction	55
3.2 The General Girsanov Transformation	58
3.3 Application to Brownian Motion	63
4 Application to Financial Economics	67
4.1 The Market Price of Risk and Risk-neutral Valuation	68
4.2 The Fundamental Pricing Rule	73
4.3 Connection with the PDE-Approach (Feynman-Kac Formula)	76

4.4	Currency Options and Siegel-Paradox	78
4.5	Change of Numeraire	79
4.6	Solution of the Siegel-Paradox	84
4.7	Admissible Strategies and Arbitrage-free Pricing	86
4.8	The “Forward Measure”	89
4.9	Option Pricing Under Stochastic Interest Rates	92
5	Term Structure Models	95
5.1	Different Descriptions of the Term Structure of Interest Rates	96
5.2	Stochastics of the Term Structure	99
5.3	The HJM-Model	102
5.4	Examples	105
5.5	The “LIBOR Market” Model	107
5.6	Caps, Floors and Swaps	111
6	Why Do We Need Itô-Calculus in Finance?	113
6.1	The Buy-Sell-Paradox	114
6.2	Local Times and Generalized Itô Formula	115
6.3	Solution of the Buy-Sell-Paradox	120
6.4	Arrow-Debreu Prices in Finance	121
6.5	The Time Value of an Option as Expected Local Time . .	123
7	Appendix: Itô Calculus Without Probabilities	125
	References	135