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Editors

Effective Computational Geometry for Curves and Surfaces

With 120 Figures and 1 Table

 Springer

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