

Marek Musiela Marek Rutkowski

Martingale Methods in Financial Modelling

Second Edition

 Springer

Table of Contents

Preface to the First Edition	V
Preface to the Second Edition	VII

Part I. Spot and Futures Markets

1. An Introduction to Financial Derivatives	3
1.1 Options	3
1.2 Futures Contracts and Options	6
1.3 Forward Contracts	7
1.4 Call and Put Spot Options	8
1.4.1 One-period Spot Market	10
1.4.2 Replicating Portfolios	11
1.4.3 Martingale Measure for a Spot Market	12
1.4.4 Absence of Arbitrage	14
1.4.5 Optimality of Replication	15
1.4.6 Put Option	18
1.5 Futures Call and Put Options	19
1.5.1 Futures Contracts and Futures Prices	20
1.5.2 One-period Futures Market	20
1.5.3 Martingale Measure for a Futures Market	22
1.5.4 Absence of Arbitrage	22
1.5.5 One-period Spot/Futures Market	24
1.6 Forward Contracts	25
1.6.1 Forward Price	25
1.7 Options of American Style	27
1.8 Universal No-arbitrage Inequalities	32
2. Discrete-time Security Markets	35
2.1 The Cox-Ross-Rubinstein Model	36
2.1.1 Binomial Lattice for the Stock Price	36
2.1.2 Recursive Pricing Procedure	38
2.1.3 CRR Option Pricing Formula	43

2.2	Martingale Properties of the CRR Model	46
2.2.1	Martingale Measures	47
2.2.2	Risk-neutral Valuation Formula	50
2.3	The Black-Scholes Option Pricing Formula	51
2.4	Valuation of American Options	56
2.4.1	American Call Options	56
2.4.2	American Put Options	58
2.4.3	American Claim	60
2.5	Options on a Dividend-paying Stock	61
2.6	Finite Spot Markets	63
2.6.1	Self-financing Trading Strategies	63
2.6.2	Arbitrage Opportunities	65
2.6.3	Arbitrage Price	66
2.6.4	Risk-neutral Valuation Formula	67
2.6.5	Price Systems	70
2.6.6	Completeness of a Finite Market	73
2.6.7	Change of a Numeraire	74
2.7	Finite Futures Markets	75
2.7.1	Self-financing Futures Strategies	75
2.7.2	Martingale Measures for a Futures Market	77
2.7.3	Risk-neutral Valuation Formula	79
2.8	Futures Prices Versus Forward Prices	79
2.9	Discrete-time Models with Infinite State Space	82
3.	Benchmark Models in Continuous Time	83
3.1	The Black-Scholes Model	84
3.1.1	Risk-free Bond	84
3.1.2	Stock Price	84
3.1.3	Self-financing Trading Strategies	88
3.1.4	Martingale Measure for the Spot Market	89
3.1.5	Black-Scholes Option Pricing Formula	93
3.1.6	Case of Time-dependent Coefficients	100
3.1.7	Merton's Model	101
3.1.8	Put-Call Parity for Spot Options	102
3.1.9	Black-Scholes PDE	103
3.1.10	A Riskless Portfolio Method	105
3.1.11	Black-Scholes Sensitivities	108
3.1.12	Market Imperfections	113
3.1.13	Numerical Methods	114
3.2	A Dividend-paying Stock	116
3.2.1	Case of a Constant Dividend Yield	116
3.2.2	Case of Known Dividends	118
3.3	Bachelier Model	122
3.3.1	Bachelier Option Pricing Formula	123
3.3.2	Bachelier's PDE	124

3.3.3	Bachelier Sensitivities	125
3.4	Black Model	126
3.4.1	Self-financing Futures Strategies	127
3.4.2	Martingale Measure for the Futures Market	127
3.4.3	Black's Futures Option Formula	128
3.4.4	Options on Forward Contracts	132
3.4.5	Forward and Futures Prices	134
3.5	Robustness of the Black-Scholes Approach	135
3.5.1	Uncertain Volatility	135
3.5.2	European Call and Put Options	136
3.5.3	Convex Path-independent European Claims	139
3.5.4	General Path-independent European Claims	144
4.	Foreign Market Derivatives	147
4.1	Cross-currency Market Model	147
4.1.1	Domestic Martingale Measure	148
4.1.2	Foreign Martingale Measure	150
4.1.3	Foreign Stock Price Dynamics	151
4.2	Currency Forward Contracts and Options	152
4.2.1	Forward Exchange Rate	153
4.2.2	Currency Option Valuation Formula	154
4.3	Foreign Equity Forward Contracts	157
4.3.1	Forward Price of a Foreign Stock	157
4.3.2	Quanto Forward Contracts	159
4.4	Foreign Market Futures Contracts	160
4.5	Foreign Equity Options	164
4.5.1	Options Struck in a Foreign Currency	164
4.5.2	Options Struck in Domestic Currency	166
4.5.3	Quanto Options	167
4.5.4	Equity-linked Foreign Exchange Options	169
5.	American Options	171
5.1	Valuation of American Claims	172
5.2	American Call and Put Options	180
5.3	Early Exercise Representation of an American Put	182
5.4	Analytical Approach	185
5.5	Approximations of the American Put Price	188
5.6	Option on a Dividend-paying Stock	191
6.	Exotic Options	193
6.1	Packages	194
6.2	Forward-start Options	195
6.3	Chooser Options	196
6.4	Compound Options	197
6.5	Digital Options	198

6.6	Barrier Options	199
6.7	Lookback Options	202
6.8	Asian Options	206
6.9	Basket Options	209
6.10	Quantile Options	213
6.11	Other Exotic Options	216
7.	Volatility Risk	217
7.1	Implied Volatilities of Traded Options	219
7.1.1	Historical Volatility	219
7.1.2	Implied Volatility	220
7.1.3	Implied Volatility Versus Historical Volatility	221
7.1.4	Approximate Formulas	222
7.1.5	Implied Volatility Surface	223
7.1.6	Asymptotic Behavior of the Implied Volatility	226
7.1.7	Marked-to-Market Models	229
7.1.8	Vega Hedging	230
7.1.9	Correlated Brownian Motions	232
7.1.10	Forward-start Options	234
7.2	Extensions of the Black-Scholes Model	237
7.2.1	CEV Model	237
7.2.2	Shifted Lognormal Models	241
7.3	Local Volatility Models	242
7.3.1	Implied Risk-Neutral Probability Law	242
7.3.2	Local Volatility	245
7.3.3	Mixture Models	251
7.3.4	Advantages and Drawbacks of LV Models	254
7.4	Stochastic Volatility Models	255
7.4.1	PDE Approach	256
7.4.2	Examples of SV Models	257
7.4.3	Hull and White Model	258
7.4.4	Heston's Model	263
7.4.5	SABR Model	265
7.5	Dynamical Models of Volatility Surfaces	267
7.5.1	Dynamics of the Local Volatility Surface	267
7.5.2	Dynamics of the Implied Volatility Surface	268
7.6	Alternative Approaches	272
7.6.1	Modelling of Asset Returns	272
7.6.2	Modelling of Volatility and Realized Variance	277
8.	Continuous-time Security Markets	279
8.1	Standard Market Models	280
8.1.1	Standard Spot Market	280
8.1.2	Futures Market	289
8.1.3	Choice of a Numeraire	291

8.1.4	Existence of a Martingale Measure.....	295
8.1.5	Fundamental Theorem of Asset Pricing	296
8.2	Multidimensional Black-Scholes Model	298
8.2.1	Market Completeness	300
8.2.2	Variance-minimizing Hedging	302
8.2.3	Risk-minimizing Hedging	303
8.2.4	Market Imperfections	310

Part II. Fixed-income Markets

9.	Interest Rates and Related Contracts.....	315
9.1	Zero-coupon Bonds	315
9.1.1	Term Structure of Interest Rates	316
9.1.2	Forward Interest Rates	317
9.1.3	Short-term Interest Rate	318
9.2	Coupon-bearing Bonds	318
9.2.1	Yield-to-Maturity	319
9.2.2	Market Conventions	321
9.3	Interest Rate Futures	322
9.3.1	Treasury Bond Futures	322
9.3.2	Bond Options	324
9.3.3	Treasury Bill Futures	324
9.3.4	Eurodollar Futures	326
9.4	Interest Rate Swaps	327
9.4.1	Forward Rate Agreements	328
9.5	Stochastic Models of Bond Prices.....	331
9.5.1	Arbitrage-free Family of Bond Prices.....	331
9.5.2	Expectations Hypotheses	332
9.5.3	Case of Itô Processes	333
9.5.4	Market Price for Interest Rate Risk.....	336
9.6	Forward Measure Approach.....	337
9.6.1	Forward Price.....	339
9.6.2	Forward Martingale Measure.....	340
9.6.3	Forward Processes	343
9.6.4	Choice of a Numeraire	344
10.	Short-Term Rate Models	347
10.1	Single-factor Models	348
10.1.1	Time-homogeneous Models	348
10.1.2	Time-inhomogeneous Models.....	359
10.1.3	Model Choice	363
10.1.4	American Bond Options	365
10.1.5	Options on Coupon-bearing Bonds	366
10.2	Multi-factor Models.....	367

10.2.1	State Variables	367
10.2.2	Affine Models	368
10.2.3	Yield Models	369
10.3	Extended CIR Model	371
10.3.1	Squared Bessel Process	371
10.3.2	Model Construction	372
10.3.3	Change of a Probability Measure	372
10.3.4	Zero-coupon Bond	373
10.3.5	Case of Constant Coefficients	375
10.3.6	Case of Piecewise Constant Coefficients	375
10.3.7	Dynamics of Zero-coupon Bond	377
10.3.8	Transition Densities	378
10.3.9	Bond Option	380
11.	Models of Instantaneous Forward Rates	381
11.1	Heath-Jarrow-Morton Methodology	382
11.1.1	Ho and Lee Model	383
11.1.2	Heath-Jarrow-Morton Model	384
11.1.3	Absence of Arbitrage	386
11.1.4	Short-term Interest Rate	391
11.2	Gaussian HJM Model	392
11.2.1	Markovian Case	394
11.3	European Spot Options	398
11.3.1	Bond Options	399
11.3.2	Stock Options	402
11.3.3	Option on a Coupon-bearing Bond	405
11.3.4	Pricing of General Contingent Claims	408
11.3.5	Replication of Options	410
11.4	Volatilities and Correlations	413
11.4.1	Volatilities	413
11.4.2	Correlations	415
11.5	Futures Price	416
11.5.1	Futures Options	417
11.6	PDE Approach to Interest Rate Derivatives	421
11.6.1	PDEs for Spot Derivatives	421
11.6.2	PDEs for Futures Derivatives	425
11.7	Recent Developments	429
12.	Market LIBOR Models	431
12.1	Forward and Futures LIBORs	433
12.1.1	One-period Swap Settled in Arrears	433
12.1.2	One-period Swap Settled in Advance	435
12.1.3	Eurodollar Futures	436
12.1.4	LIBOR in the Gaussian HJM Model	437
12.2	Interest Rate Caps and Floors	439

12.3	Valuation in the Gaussian HJM Model	441
12.3.1	Plain-vanilla Caps and Floors	441
12.3.2	Exotic Caps	443
12.3.3	Captions	445
12.4	LIBOR Market Models	446
12.4.1	Black's Formula for Caps	446
12.4.2	Miltersen, Sandmann and Sondermann Approach	448
12.4.3	Brace, Gatarek and Musiela Approach	448
12.4.4	Musiela and Rutkowski Approach	451
12.4.5	Jamshidian's Approach	455
12.5	Properties of the Lognormal LIBOR Model	458
12.5.1	Transition Density of the LIBOR	459
12.5.2	Transition Density of the Forward Bond Price	461
12.6	Valuation in the Lognormal LIBOR Model	464
12.6.1	Pricing of Caps and Floors	464
12.6.2	Hedging of Caps and Floors	466
12.6.3	Valuation of European Claims	468
12.6.4	Bond Options	471
12.7	Extensions of the LLM Model	473
13.	Alternative Market Models	475
13.1	Swaps and Swaptions	476
13.1.1	Forward Swap Rates	476
13.1.2	Swaptions	480
13.1.3	Exotic Swap Derivatives	482
13.2	Valuation in the Gaussian HJM Model	485
13.2.1	Swaptions	485
13.2.2	CMS Spread Options	485
13.2.3	Yield Curve Swaps	487
13.3	Co-terminal Swap Rates	488
13.3.1	Jamshidian's Approach	493
13.3.2	Valuation of Co-terminal Swaptions	494
13.3.3	Hedging of Swaptions	495
13.3.4	Bermudan Swaptions	496
13.4	Co-initial Swap Rates	497
13.4.1	Valuation of Co-initial Swaptions	500
13.4.2	Valuation of Exotic Options	501
13.5	Co-sliding Swap Rates	502
13.5.1	Modelling of Co-sliding Swap Rates	502
13.5.2	Valuation of Co-sliding Swaptions	506
13.6	Swap Rate Model Versus LIBOR Model	508
13.6.1	Swaptions in the LLM Model	509
13.6.2	Caplets in the Co-terminal Swap Market Model	513
13.7	Markov-functional Models	514
13.7.1	Terminal Swap Rate Model	515

13.7.2 Calibration of Markov-functional Models	517
13.8 Flesaker and Hughston Approach	521
13.8.1 Rational Lognormal Model	523
13.8.2 Valuation of Caps and Swaptions	524
14. Cross-currency Derivatives	527
14.1 Arbitrage-free Cross-currency Markets	528
14.1.1 Forward Price of a Foreign Asset	530
14.1.2 Valuation of Foreign Contingent Claims	534
14.1.3 Cross-currency Rates	535
14.2 Gaussian Model	535
14.2.1 Currency Options	536
14.2.2 Foreign Equity Options	537
14.2.3 Cross-currency Swaps	542
14.2.4 Cross-currency Swaptions	553
14.2.5 Basket Caps	556
14.3 Model of Forward LIBOR Rates	557
14.3.1 Quanto Cap	558
14.3.2 Cross-currency Swap	560
14.4 Concluding Remarks	561

Part III. APPENDICES

A. Conditional Expectations	565
B. Itô Stochastic Calculus	569
B.1 Itô Integral	569
B.2 Girsanov's Theorem	576
B.3 Itô-Tanaka-Meyer Formula	578
B.4 Laws of Certain Functionals of a Brownian Motion	579
References	583
Index	629