H. Ehrig \cdot K. Ehrig U. Prange \cdot G. Taentzer

Fundamentals of Algebraic Graph Transformation

With 41 Figures



Contents

Part I Introduction to Graph Transformation Systems					
1	General Introduction				
	1.1	Gener	al Overview of Graph Grammars and Graph		
		Trans	formation	5	
		1.1.1	What Is Graph Transformation?	6	
		1.1.2	Aims and Paradigms of Graph Transformation	6	
		1.1.3	Overview of Various Approaches	9	
	1.2	The N	Main Ideas of the Algebraic Graph Transformation		
		Appro	oach	10	
		1.2.1	The DPO Approach	11	
		1.2.2	The Algebraic Roots	12	
		1.2.3	From the DPO to the SPO Approach	13	
		1.2.4	From Graphs to High-Level Structures	14	
	1.3	The C	Chapters of This Book and the Main Results	15	
		1.3.1	Part I: Introduction to Graph Transformation Systems .	15	
		1.3.2	Part II: Adhesive HLR Categories and Systems	15	
		1.3.3	Part III: Typed Attributed Graph Transformation		
			Systems	16	
		1.3.4	Part IV: Case Study and Tool Support	17	
		1.3.5	Appendices	17	
		1.3.6	Hints for Reading This Book	17	
	1.4	Biblio	graphic Notes and Further Topics	17	
		1.4.1	Concepts of Graph Grammars and Graph		
			Transformation Systems	17	
		1.4.2	Application Areas of Graph Transformation Systems	19	
		1.4.3	Languages and Tools for Graph Transformation Systems	19	
		1.4.4	Future Work	20	

X	Contents

2	Gra	phs, Typed Graphs, and the Gluing Construction 2	1
	2.1	Graphs and Typed Graphs	1
	2.2	Introduction to Categories	5
	2.3		9
	2.4	Pullbacks as the Dual Construction of Pushouts	3
3	Gra	ph Transformation Systems 3	
	3.1	Basic Definitions for GT Systems	
	3.2	<u> </u>	3
	3.3	Local Church–Rosser and Parallelism Theorems for GT	_
		V	7
	3.4		3
		U	4
		3.4.3 Confluence, Local Confluence, Termination, and	6
			9
		3.4.4 Functional Behavior of GT Systems and Termination	
		J	2
	3.5	Graph Constraints and Application Conditions 6	4
– Pa	rt II	Adhesive High-Level Replacement Categories and System	_ s
		Carry March 1988	_
4	Ad	•	7
4	4.1	Van Kampen Squares and Adhesive Categories	7
4	$\frac{4.1}{4.2}$	Van Kampen Squares and Adhesive Categories	7 86
4	4.1	Van Kampen Squares and Adhesive Categories	
4 5	4.1 4.2 4.3	Van Kampen Squares and Adhesive Categories	7 6 6
-	4.1 4.2 4.3	Van Kampen Squares and Adhesive Categories 7 Adhesive HLR Categories 8 HLR Properties of Adhesive HLR Categories 9 hesive High-Level Replacement Systems 10 Basic Concepts of Adhesive HLR Systems 10	77 86 96 91
-	4.1 4.2 4.3 Ad	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems10	7 6 16 11 15
-	4.1 4.2 4.3 Ad l 5.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems10	7 6 10 10 10 10 10 10 10 10 10 10 10 10 10
-	4.1 4.2 4.3 Ad l 5.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems10	7 6 10 10 10 10 10 10 10 10 10 10 10 10 10
-	4.1 4.2 4.3 Ad l 5.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems10	7 6 10 10 10 10 10 10 10 10 10 10 10 10 10
-	4.1 4.2 4.3 Ad l 5.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems10	7666 11156678
-	4.1 4.2 4.3 Ad 5.1 5.2	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10	7666 11156678 8
-	4.1 4.2 4.3 Ad 5.1 5.2	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10The Local Church-Rosser and Parallelism Theorems10	7666 11156667889
-	4.1 4.2 4.3 Ad 5.1 5.2	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10	7666 11156667889
-	4.1 4.2 4.3 Ad 5.1 5.2 5.3 5.4 Em	Van Kampen Squares and Adhesive Categories 7 Adhesive HLR Categories 8 HLR Properties of Adhesive HLR Categories 9 hesive High-Level Replacement Systems 10 Basic Concepts of Adhesive HLR Systems 10 Instantiation of Adhesive HLR Systems 10 5.2.1 Graph and Typed Graph Transformation Systems 10 5.2.2 Hypergraph Transformation Systems 10 5.2.3 Petri Net Transformation Systems 10 5.2.4 Algebraic Specification Transformation Systems 10 5.2.5 Typed Attributed Graph Transformation Systems 10 The Local Church-Rosser and Parallelism Theorems 10 Concurrency Theorem and Pair Factorization 11 bedding and Local Confluence 12	766 1156 1607 1889 1785
5	4.1 4.2 4.3 Adl 5.1 5.2 5.3 5.4 Em 6.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10The Local Church-Rosser and Parallelism Theorems10Concurrency Theorem and Pair Factorization11bedding and Local Confluence12Initial Pushouts and the Gluing Condition12	766 11156678897 255
5	4.1 4.2 4.3 Adl 5.1 5.2 5.3 5.4 Em 6.1 6.2	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10The Local Church-Rosser and Parallelism Theorems10Concurrency Theorem and Pair Factorization11bedding and Local Confluence12Initial Pushouts and the Gluing Condition12Embedding and Extension Theorems13	766 11156678897 550
5	4.1 4.2 4.3 Adl 5.1 5.2 5.3 5.4 Em 6.1	Van Kampen Squares and Adhesive Categories7Adhesive HLR Categories8HLR Properties of Adhesive HLR Categories9hesive High-Level Replacement Systems10Basic Concepts of Adhesive HLR Systems10Instantiation of Adhesive HLR Systems105.2.1 Graph and Typed Graph Transformation Systems105.2.2 Hypergraph Transformation Systems105.2.3 Petri Net Transformation Systems105.2.4 Algebraic Specification Transformation Systems105.2.5 Typed Attributed Graph Transformation Systems10The Local Church-Rosser and Parallelism Theorems10Concurrency Theorem and Pair Factorization11bedding and Local Confluence12Initial Pushouts and the Gluing Condition12	766 1156678897 5500

		Contents XI			
7	Cor	nstraints and Application Conditions			
	7.1	Definition of Constraints and Application Conditions 152			
	7.2	Construction of Application Conditions from Constraints 156			
	7.3	Construction of Left from Right Application Conditions 160			
	7.4	Guaranteeing and Preservation of Constraints			
Pa	rt II	Typed Attributed Graph Transformation Systems			
8	Tvr	ped Attributed Graphs			
	8.1	Attributed Graphs and Typing			
	8.2	Pushouts as a Gluing Construction of Attributed Graphs 177			
	8.3	Pullbacks of Attributed Graphs			
9	Тур	ped Attributed Graph Transformation Systems181			
	9.1	Basic Concepts for Typed AGT Systems			
	9.2	Construction of Typed Attributed Graph Transformations 188			
	9.3	Local Church–Rosser and Parallelism Theorem for Typed			
		AGT Systems			
	9.4	Concurrency Theorem and Pair Factorization for Typed			
		AGT Systems			
		9.4.1 Pair Factorizations			
		9.4.2 Concurrency Theorem			
10	Embedding and Local Confluence for Typed AGT Systems 207				
		Embedding and Extension Theorems for Typed AGT Systems 207			
		Critical Pairs for Typed AGT Systems			
		Local Confluence Theorem for Typed AGT Systems			
11	$\mathbf{A}\mathbf{d}\mathbf{l}$	nesive HLR Categories for Typed Attributed Graphs 221			
	11.1	Attributed Graph Structure Signatures and Typed Attributed			
		Graphs			
	11.2	Definition of Concrete Adhesive HLR Categories			
	11.3	Verification of the Main Results for Typed AGT Systems 232			
12	Constraints, Application Conditions and Termination for TAGT Systems				
	12.1	Constraints and Application Conditions for Typed AGT Systems			
	19 9	Equivalence of Constraints and Application Conditions			
		Termination Criteria for Layered Typed Attributed Graph			
	12.0	Crammars 240			

13	71			
	13.1 Attributed Type Graphs with Inheritance			
	13.2 Attributed Clan Morphisms	265		
	13.3 Productions and Attributed Graph Transformation with			
	Inheritance			
	13.4 Equivalence of Concepts with and without Inheritance	278		
	t IV Case Study on Model Transformation, and ol Support by AGG			
- 14	Case Study on Model Transformation	${287}$		
	14.1 Model Transformation by Typed Attributed Graph			
	Transformation	287		
	14.2 Model Transformation from Statecharts to Petri Nets	288		
	Statecharts			
	14.2.2 Target Modeling Language: Petri Nets			
	14.2.3 Model Transformation			
	14.2.4 Termination Analysis of the Model Transformation			
	14.3 Further Case Studies			
	14.3.1 From the General Resource Model to Petri Nets			
	14.3.2 From Process Interaction Diagrams to Timed Petri Nets			
	14.4 Conclusion	304		
15	Implementation of Typed Attributed Graph			
	Transformation by AGG	305		
	15.1 Language Concepts of AGG			
	15.1.1 Graphs			
	15.1.2 Typing Facilities	306		
	15.1.3 Node and Edge Attributes	307		
	15.1.4 Rules and Matches	308		
	15.1.5 Graph Transformations	310		
	15.1.6 Graph Grammars	312		
	15.2 Analysis Techniques Implemented in AGG	312		
	15.2.1 Graph Constraints	312		
	15.2.2 Critical Pair Analysis	313		
	15.2.3 Graph Parsing	317		
	15.2.4 Termination			
	15.3 Tool Environment of AGG			
	15.3.1 Visual Environment	320		
	15.3.2 Graph Transformation Engine			
	15.3.3 Tool Integration			
	15.4 Conclusion	322		

\mathbf{A}	A Short Introduction to Category Theory	29		
	A.1 Categories			
	A.2 Construction of Categories, and Duality	30		
	A.3 Monomorphisms, Epimorphisms, and Isomorphisms	32		
	A.4 Pushouts and Pullbacks	34		
	A.5 Binary Coproducts and Initial Objects	34 0		
	A.6 Functors, Functor Categories, and Comma Categories	44		
	A.7 Isomorphism and Equivalence of Categories	50		
В	A Short Introduction to Signatures and Algebras			
	B.1 Algebraic Signatures			
	B.2 Algebras			
	B.3 Terms and Term Evaluation	557		
\mathbf{C}	Detailed Proofs			
	C.1 Completion of Proof of Fact 4.24	359		
	C.2 Proof of Lemma 6.25			
	C.3 Completion of Proof of Theorem 11.3	62		
	C.3.1 Well-Definedness	62		
	C.3.2 Functors	64		
	C.3.3 Isomorphism	65		
	C.4 Proof of Lemma 11.17	366		
	C.4.1 Well-Definedness	366		
	C.4.2 Pushout Property	67		
	C.4.3 Initial Pushout	368		
	C.5 Proof of Theorem 13.12	370		
	C.6 Proof of Lemma 13.20	73		
Re	erences	377		
Ind	ex	185		