Yoshikazu Giga

Surface Evolution Equations

A Level Set Approach

Birkhäuser Verlag Basel • Boston • Berlin

Contents

Pr	Preface							
Introduction								
1	Surface evolution equations							
	1.1	Representation of a hypersurface	15					
	1.2	Normal velocity	18					
	1.3	Curvatures	21					
	1.4	Expression of curvature tensors	26					
	1.5	Examples of surface evolution equations	33					
		1.5.1 General evolutions of isothermal interfaces	33					
		1.5.2 Evolution by principal curvatures	34					
		1.5.3 Other examples	35					
		1.5.4 Boundary conditions	35					
	1.6	Level set equations	36					
		1.6.1 Examples	36					
		1.6.2 General scaling invariance	4 0					
		1.6.3 Ellipticity	42					
		1.6.4 Geometric equations	4 6					
		1.6.5 Singularities in level set equations	49					
	1.7	Exact solutions	52					
		1.7.1 Mean curvature flow equation	52					
		1.7.2 Anisotropic version	54					
		1.7.3 Anisotropic mean curvature of the Wulff shape	58					
	•	1.7.4 Affine curvature flow equation	62					
	1.8	Notes and comments	63					
2	Visc	cosity solutions	69					
	2.1	Definitions and main expected properties	69					
		2.1.1 Definition for arbitrary functions	70					
		2.1.2 Expected properties of solutions	73					
		2.1.3 Very singular equations	77					

viii Contents

	2.2	Stability results					
		2.2.1 Remarks on a class of test functions 8					
		2.2.2 Convergence of maximum points 8					
		2.2.3 Applications					
	2.3	Boundary value problems					
	2.4	Perron's method					
		2.4.1 Closedness under supremum					
		2.4.2 Maximal subsolution					
		2.4.3 Adaptation for very singular equations					
		2.4.4 Applicability					
	2.5	Notes and comments					
3	Con	omparison principle 109					
J	3.1	Typical statements					
	0.1	3.1.1 Bounded domains					
		3.1.2 General domains					
		3.1.3 Applicability					
	3.2	Alternate definition of viscosity solutions					
	3.2	3.2.1 Definition involving semijets					
		3.2.2 Solutions on semiclosed time intervals					
	3.3	General idea for the proof of comparison principles					
	ა.ა	· · · · · · · · · · · · · · · · · · ·					
		3.3.1 A typical problem					
	9.4						
	3.4	Proof of comparison principles for parabolic equations					
	9 5						
	3.5	Lipschitz preserving and convexity preserving properties					
	3.6	Spatially inhomogeneous equations					
		3.6.1 Inhomogeneity in first order perturbation					
	0.7	3.6.2 Inhomogeneity in higher order terms					
	3.7	Boundary value problems					
	3.8	Notes and comments					
4	Clas	ssical level set method 16					
ंद्रा	4.1	Brief sketch of a level set method					
	4.2	Uniqueness of bounded evolutions					
		4.2.1 Invariance under change of dependent variables 16					
		4.2.2 Orientation-free surface evolution equations 17					
		4.2.3 Uniqueness					
		4.2.4 Unbounded evolutions					
	4.3	Existence by Perron's method					
	4.4	Existence by approximation					
	4.5	Various properties of evolutions					
	46	Convergence properties for level set equations 19					

Contents

	4.7	7 Instant extinction						
	4.8	Notes	and comments	201				
5	Set-	theoret	ic approach	207				
	5.1	Set-th	eoretic solutions	207				
		5.1.1	Definition and its characterization	208				
		5.1.2	Characterization of solutions of level set equations					
		5.1.3	Characterization by distance functions					
		5.1.4	Comparison principle for sets					
		5.1.5	Convergence of sets and functions					
	5.2	Level	set solutions					
		5.2.1	Nonuniqueness					
		5.2.2	Definition of level set solutions					
		5.2.3	Uniqueness of level set solutions					
	5.3	Barrie	er solutions					
	5.4		stency					
	0.1	5.4.1	Nested family of subsolutions					
		5.4.2	Applications					
		5.4.3	Relation among various solutions					
	5.5		ation and comparison principle					
	5.6		and comments					
	5.0	Notes	and comments	200				
Bibliography Notation Index								