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Information and Self-Organization

A Macroscopic Approach to Complex Systems

Third Enlarged Edition

With 66 Figures

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