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Quantum Mechanics

Fourth Edition

With 123 Figures, 16 Tables,

Numerous Worked Examples and 127 Problems



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