

# Contents

<b>1. Introduction</b> . . . . .	1
1.1 Computational Fluid Dynamics . . . . .	1
1.2 Levels of Approximation: General . . . . .	2
1.3 Statement of the Scale Separation Problem . . . . .	3
1.4 Usual Levels of Approximation . . . . .	5
1.5 Large-Eddy Simulation: from Practice to Theory. Structure of the Book . . . . .	9
<b>2. Formal Introduction to Scale Separation: Band-Pass Filtering</b> . . . . .	15
2.1 Definition and Properties of the Filter in the Homogeneous Case . . . . .	15
2.1.1 Definition . . . . .	15
2.1.2 Fundamental Properties . . . . .	17
2.1.3 Characterization of Different Approximations . . . . .	18
2.1.4 Differential Filters . . . . .	20
2.1.5 Three Classical Filters for Large-Eddy Simulation . . . . .	21
2.1.6 Differential Interpretation of the Filters . . . . .	26
2.2 Spatial Filtering: Extension to the Inhomogeneous Case . . . . .	31
2.2.1 General . . . . .	31
2.2.2 Non-uniform Filtering Over an Arbitrary Domain . . . . .	32
2.2.3 Local Spectrum of Commutation Error . . . . .	42
2.3 Time Filtering: a Few Properties . . . . .	43
<b>3. Application to Navier–Stokes Equations</b> . . . . .	45
3.1 Navier–Stokes Equations . . . . .	46
3.1.1 Formulation in Physical Space . . . . .	46
3.1.2 Formulation in General Coordinates . . . . .	46
3.1.3 Formulation in Spectral Space . . . . .	47
3.2 Filtered Navier–Stokes Equations in Cartesian Coordinates (Homogeneous Case) . . . . .	48
3.2.1 Formulation in Physical Space . . . . .	48
3.2.2 Formulation in Spectral Space . . . . .	48

3.3	Decomposition of the Non-linear Term.	
	Associated Equations for the Conventional Approach . . . . .	49
3.3.1	Leonard's Decomposition . . . . .	49
3.3.2	Germano Consistent Decomposition . . . . .	59
3.3.3	Germano Identity . . . . .	61
3.3.4	Invariance Properties . . . . .	64
3.3.5	Realizability Conditions . . . . .	72
3.4	Extension to the Inhomogeneous Case	
	for the Conventional Approach . . . . .	74
3.4.1	Second-Order Commuting Filter . . . . .	74
3.4.2	High-Order Commuting Filters . . . . .	77
3.5	Filtered Navier–Stokes Equations in General Coordinates . . . . .	77
3.5.1	Basic Form of the Filtered Equations . . . . .	77
3.5.2	Simplified Form of the Equations –	
	Non-linear Terms Decomposition . . . . .	78
3.6	Closure Problem . . . . .	78
3.6.1	Statement of the Problem . . . . .	78
3.6.2	Postulates . . . . .	79
3.6.3	Functional and Structural Modeling . . . . .	80
<b>4.</b>	<b>Other Mathematical Models for the Large-Eddy</b>	
	<b>Simulation Problem . . . . .</b>	<b>83</b>
4.1	Ensemble-Averaged Models . . . . .	83
4.1.1	Yoshizawa's Partial Statistical Average Model . . . . .	83
4.1.2	McComb's Conditional Mode Elimination Procedure . . . . .	84
4.2	Regularized Navier–Stokes Models . . . . .	85
4.2.1	Leray's Model . . . . .	86
4.2.2	Holm's Navier–Stokes- $\alpha$ Model . . . . .	86
4.2.3	Ladyzenskaja's Model . . . . .	89
<b>5.</b>	<b>Functional Modeling (Isotropic Case) . . . . .</b>	<b>91</b>
5.1	Phenomenology of Inter-Scale Interactions . . . . .	91
5.1.1	Local Isotropy Assumption: Consequences . . . . .	92
5.1.2	Interactions Between Resolved and Subgrid Scales . . . . .	93
5.1.3	A View in Physical Space . . . . .	102
5.1.4	Summary . . . . .	104
5.2	Basic Functional Modeling Hypothesis . . . . .	104
5.3	Modeling of the Forward Energy Cascade Process . . . . .	105
5.3.1	Spectral Models . . . . .	105
5.3.2	Physical Space Models . . . . .	109
5.3.3	Improvement of Models in the Physical Space . . . . .	133
5.3.4	Implicit Diffusion: the ILES Concept . . . . .	161
5.4	Modeling the Backward Energy Cascade Process . . . . .	171
5.4.1	Preliminary Remarks . . . . .	171

5.4.2	Deterministic Statistical Models . . . . .	172
5.4.3	Stochastic Models . . . . .	178
<b>6.</b>	<b>Functional Modeling:</b>	
	<b>Extension to Anisotropic Cases . . . . .</b>	<b>187</b>
6.1	Statement of the Problem . . . . .	187
6.2	Application of Anisotropic Filter to Isotropic Flow . . . . .	187
6.2.1	Scalar Models . . . . .	188
6.2.2	Batten's Mixed Space-Time Scalar Estimator . . . . .	191
6.2.3	Tensorial Models . . . . .	191
6.3	Application of an Isotropic Filter to a Shear Flow . . . . .	193
6.3.1	Phenomenology of Inter-Scale Interactions . . . . .	193
6.3.2	Anisotropic Models: Scalar Subgrid Viscosities . . . . .	198
6.3.3	Anisotropic Models: Tensorial Subgrid Viscosities . . . . .	202
6.4	Remarks on Flows Submitted to Strong Rotation Effects . . . . .	208
<b>7.</b>	<b>Structural Modeling . . . . .</b>	<b>209</b>
7.1	Introduction and Motivations . . . . .	209
7.2	Formal Series Expansions . . . . .	210
7.2.1	Models Based on Approximate Deconvolution . . . . .	210
7.2.2	Non-linear Models . . . . .	223
7.2.3	Homogenization-Technique-Based Models . . . . .	228
7.3	Scale Similarity Hypotheses and Models Using Them . . . . .	231
7.3.1	Scale Similarity Hypotheses . . . . .	231
7.3.2	Scale Similarity Models . . . . .	232
7.3.3	A Bridge Between Scale Similarity and Approximate Deconvolution Models. Generalized Similarity Models . . . . .	236
7.4	Mixed Modeling . . . . .	237
7.4.1	Motivations . . . . .	237
7.4.2	Examples of Mixed Models . . . . .	239
7.5	Differential Subgrid Stress Models . . . . .	243
7.5.1	Deardorff Model . . . . .	243
7.5.2	Fureby Differential Subgrid Stress Model . . . . .	244
7.5.3	Velocity-Filtered-Density-Function-Based Subgrid Stress Models . . . . .	245
7.5.4	Link with the Subgrid Viscosity Models . . . . .	248
7.6	Stretched-Vortex Subgrid Stress Models . . . . .	249
7.6.1	General . . . . .	249
7.6.2	S3/S2 Alignment Model . . . . .	250
7.6.3	S3/ $\omega$ Alignment Model . . . . .	250
7.6.4	Kinematic Model . . . . .	250
7.7	Explicit Evaluation of Subgrid Scales . . . . .	251
7.7.1	Fractal Interpolation Procedure . . . . .	253
7.7.2	Chaotic Map Model . . . . .	254

7.7.3	Kerstein's ODT-Based Method . . . . .	257
7.7.4	Kinematic-Simulation-Based Reconstruction . . . . .	259
7.7.5	Velocity Filtered Density Function Approach . . . . .	260
7.7.6	Subgrid Scale Estimation Procedure . . . . .	261
7.7.7	Multi-level Simulations . . . . .	263
7.8	Direct Identification of Subgrid Terms . . . . .	272
7.8.1	Linear-Stochastic-Estimation-Based Model . . . . .	274
7.8.2	Neural-Network-Based Model . . . . .	275
7.9	Implicit Structural Models . . . . .	275
7.9.1	Local Average Method . . . . .	276
7.9.2	Scale Residual Model . . . . .	278
<b>8.</b>	<b>Numerical Solution: Interpretation and Problems . . . . .</b>	<b>281</b>
8.1	Dynamic Interpretation of the Large-Eddy Simulation . . . . .	281
8.1.1	Static and Dynamic Interpretations: Effective Filter . . . . .	281
8.1.2	Theoretical Analysis of the Turbulence Generated by Large-Eddy Simulation . . . . .	283
8.2	Ties Between the Filter and Computational Grid. Pre-filtering . . . . .	288
8.3	Numerical Errors and Subgrid Terms . . . . .	290
8.3.1	Ghosal's General Analysis . . . . .	290
8.3.2	Pre-filtering Effect . . . . .	294
8.3.3	Conclusions . . . . .	297
8.3.4	Remarks on the Use of Artificial Dissipations . . . . .	299
8.3.5	Remarks Concerning the Time Integration Method . . . . .	303
<b>9.</b>	<b>Analysis and Validation of Large-Eddy Simulation Data . . . . .</b>	<b>305</b>
9.1	Statement of the Problem . . . . .	305
9.1.1	Type of Information Contained in a Large-Eddy Simulation . . . . .	305
9.1.2	Validation Methods . . . . .	306
9.1.3	Statistical Equivalency Classes of Realizations . . . . .	307
9.1.4	Ideal LES and Optimal LES . . . . .	310
9.1.5	Mathematical Analysis of Sensitivities and Uncertainties in Large-Eddy Simulation . . . . .	311
9.2	Correction Techniques . . . . .	313
9.2.1	Filtering the Reference Data . . . . .	313
9.2.2	Evaluation of Subgrid-Scale Contribution . . . . .	314
9.2.3	Evaluation of Subgrid-Scale Kinetic Energy . . . . .	315
9.3	Practical Experience . . . . .	318
<b>10.</b>	<b>Boundary Conditions . . . . .</b>	<b>323</b>
10.1	General Problem . . . . .	323
10.1.1	Mathematical Aspects . . . . .	323
10.1.2	Physical Aspects . . . . .	324

10.2	Solid Walls . . . . .	326
10.2.1	Statement of the Problem . . . . .	326
10.2.2	A Few Wall Models . . . . .	332
10.2.3	Wall Models: Achievements and Problems . . . . .	351
10.3	Case of the Inflow Conditions . . . . .	354
10.3.1	Required Conditions . . . . .	354
10.3.2	Inflow Condition Generation Techniques . . . . .	354
<b>11.</b>	<b>Coupling Large-Eddy Simulation</b>	
	<b>with Multiresolution/Multidomain Techniques . . . . .</b>	<b>369</b>
11.1	Statement of the Problem . . . . .	369
11.2	Methods with Full Overlap . . . . .	371
11.2.1	One-Way Coupling Algorithm . . . . .	372
11.2.2	Two-Way Coupling Algorithm . . . . .	372
11.2.3	FAS-like Multilevel Method . . . . .	373
11.2.4	Kravchenko et al. Method . . . . .	374
11.3	Methods Without Full Overlap . . . . .	376
11.4	Coupling Large-Eddy Simulation with Adaptive	
	Mesh Refinement . . . . .	377
11.4.1	Statement of the Problem . . . . .	377
11.4.2	Error Estimation . . . . .	378
<b>12.</b>	<b>Hybrid RANS/LES Approaches . . . . .</b>	<b>383</b>
12.1	Motivations and Presentation . . . . .	383
12.2	Zonal Decomposition . . . . .	384
12.2.1	Statement of the Problem . . . . .	384
12.2.2	Sharp Transition . . . . .	385
12.2.3	Smooth Transition . . . . .	387
12.2.4	Zonal RANS/LES Approach as Wall Model . . . . .	388
12.3	Nonlinear Disturbance Equations . . . . .	390
12.4	Universal Modeling . . . . .	391
12.4.1	Germano's Hybrid Model . . . . .	392
12.4.2	Speziale's Rescaling Method and Related Approaches . . . . .	393
12.4.3	Baurle's Blending Strategy . . . . .	394
12.4.4	Arunajatesan's Modified Two-Equation Model . . . . .	396
12.4.5	Bush-Mani Limiters . . . . .	397
12.4.6	Magagnato's Two-Equation Model . . . . .	398
12.5	Toward a Theoretical Status for Hybrid	
	RANS/LES Approaches . . . . .	399
<b>13.</b>	<b>Implementation . . . . .</b>	<b>401</b>
13.1	Filter Identification. Computing the Cutoff Length . . . . .	401
13.2	Explicit Discrete Filters . . . . .	404
13.2.1	Uniform One-Dimensional Grid Case . . . . .	404
13.2.2	Extension to the Multi-Dimensional Case . . . . .	407

13.2.3	Extension to the General Case. Convolution Filters . . .	407
13.2.4	High-Order Elliptic Filters . . . . .	408
13.3	Implementation of the Structure Function Models . . . . .	408
<b>14.</b>	<b>Examples of Applications . . . . .</b>	<b>411</b>
14.1	Homogeneous Turbulence . . . . .	411
14.1.1	Isotropic Homogeneous Turbulence . . . . .	411
14.1.2	Anisotropic Homogeneous Turbulence . . . . .	412
14.2	Flows Possessing a Direction of Inhomogeneity . . . . .	414
14.2.1	Time-Evolving Plane Channel . . . . .	414
14.2.2	Other Flows . . . . .	418
14.3	Flows Having at Most One Direction of Homogeneity . . . . .	419
14.3.1	Round Jet . . . . .	419
14.3.2	Backward Facing Step . . . . .	426
14.3.3	Square-Section Cylinder . . . . .	430
14.3.4	Other Examples . . . . .	431
14.4	Industrial Applications . . . . .	432
14.4.1	Large-Eddy Simulation for Nuclear Power Plants . . . . .	432
14.4.2	Flow in a Mixed-Flow Pump . . . . .	435
14.4.3	Flow Around a Landing Gear Configuration . . . . .	437
14.4.4	Flow Around a Full-Scale Car . . . . .	437
14.5	Lessons . . . . .	439
14.5.1	General Lessons . . . . .	439
14.5.2	Subgrid Model Efficiency . . . . .	442
14.5.3	Wall Model Efficiency . . . . .	444
14.5.4	Mesh Generation for <i>Building Blocks</i> Flows . . . . .	445
<b>15.</b>	<b>Coupling with Passive/Active Scalar . . . . .</b>	<b>449</b>
15.1	Scope of this Chapter . . . . .	449
15.2	The Passive Scalar Case . . . . .	450
15.2.1	Physical Model . . . . .	450
15.2.2	Dynamics of the Passive Scalar . . . . .	453
15.2.3	Extensions of Functional Models . . . . .	461
15.2.4	Extensions of Structural Models . . . . .	466
15.2.5	Generalized Subgrid Modeling for Arbitrary Non-linear Functions of an Advected Scalar . . . . .	468
15.2.6	Models for Subgrid Scalar Variance and Scalar Subgrid Mixing Rate . . . . .	469
15.2.7	A Few Applications . . . . .	472
15.3	The Active Scalar Case: Stratification and Buoyancy Effects . . . . .	472
15.3.1	Physical Model . . . . .	472
15.3.2	Some Insights into the Active Scalar Dynamics . . . . .	474
15.3.3	Extensions of Functional Models . . . . .	481
15.3.4	Extensions of Structural Models . . . . .	487
15.3.5	Subgrid Kinetic Energy Estimates . . . . .	490

15.3.6 More Complex Physical Models .....	492
15.3.7 A Few Applications .....	492
<b>A. Statistical and Spectral Analysis of Turbulence .....</b>	<b>495</b>
A.1 Turbulence Properties .....	495
A.2 Foundations of the Statistical Analysis of Turbulence .....	495
A.2.1 Motivations .....	495
A.2.2 Statistical Average: Definition and Properties .....	496
A.2.3 Ergodicity Principle .....	496
A.2.4 Decomposition of a Turbulent Field .....	498
A.2.5 Isotropic Homogeneous Turbulence .....	499
A.3 Introduction to Spectral Analysis of the Isotropic Turbulent Fields .....	499
A.3.1 Definitions .....	499
A.3.2 Modal Interactions .....	501
A.3.3 Spectral Equations .....	502
A.4 Characteristic Scales of Turbulence .....	504
A.5 Spectral Dynamics of Isotropic Homogeneous Turbulence ....	504
A.5.1 Energy Cascade and Local Isotropy .....	504
A.5.2 Equilibrium Spectrum .....	505
<b>B. EDQNM Modeling .....</b>	<b>507</b>
B.1 Isotropic EDQNM Model .....	507
B.2 Cambon's Anisotropic EDQNM Model .....	509
B.3 EDQNM Model for Isotropic Passive Scalar .....	511
<b>Bibliography .....</b>	<b>513</b>
<b>Index .....</b>	<b>553</b>