

W. Horsthemke R. Lefever

Noise-Induced Transitions

Theory and Applications
in Physics, Chemistry, and Biology

With 56 Figures

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

Contents

1. Introduction	1
1.1 Deterministic and Random Aspects of Macroscopic Order	1
1.2 From Crystals to Dissipative Structures	6
1.2.1 Macroscopic Description of Self-Organization in a Constant Environment	7
1.2.2 Internal Fluctuations	13
1.3 External Noise	14
1.4 Noise-Induced Nonequilibrium Phase Transitions	15
1.5 Modeling Environmental Fluctuations	16
2. Elements of Probability Theory	23
2.1 Probability Triple and Random Variables	23
2.1.1 The Sample Space Ω and the Field of Events \mathcal{A}	23
2.1.2 Random Variables	25
2.1.3 The Probability Measure P	27
2.1.4 The Distribution Function	28
2.1.5 Moments and Extrema	29
2.1.6 Joint Random Variables	34
2.1.7 Conditional Probabilities	35
2.2 Stochastic Processes	40
2.2.1 Definitions	40
2.2.2 Separability	42
2.2.3 Continuity	42
2.2.4 Stationarity	43
2.3 Brownian Motion: The Wiener Process	44
2.4 Brownian Motion: The Ornstein-Uhlenbeck Process	49
2.5 The Poisson Process	53
3. Stochastic Models of Environmental Fluctuations	55
3.1 Correlation Function and Noise Spectrum	55
3.2 The White-Noise Process	59
4. Markovian Diffusion Processes	65
4.1 Markovian Processes: Definition	65
4.2 Markovian Diffusion Processes: Definition	69
4.3 The Ornstein-Uhlenbeck Process Revisited and Doob's Theorem ..	72
4.4 The Kolmogorov Backward Equation and the Fokker-Planck Equation	73

4.5	Pawula's Theorem	78
4.6	Non-Gaussian White Noise	81
5.	Stochastic Differential Equations	82
5.1	Stochastic Integrals: A First Encounter	82
5.2	The Ito Integral	88
5.3	Ito Stochastic Differential Equations and Diffusion Processes	92
5.3.1	Existence and Uniqueness of Solutions	93
5.3.2	Markov Property of Solutions	94
5.3.3	Ito Equations and the Fokker-Planck Equation	95
5.4	Stratonovich Stochastic Integral	97
5.4.1	Definition of the Stratonovich Integral and Its Relation with the Ito Integral	98
5.4.2	Ito or Stratonovich: A Guide for the Perplexed Modeler	101
5.5	Classification of the Boundaries of a Diffusion Process	104
6.	Noise-Induced Nonequilibrium Phase Transitions	108
6.1	Stationary Solution of the Fokker-Planck Equation	109
6.2	The Neighborhood of Deterministic Behavior: Additive and Small Multiplicative Noise	114
6.3	Transition Phenomena in a Fluctuating Environment	118
6.4	The Verhulst System in a White-Noise Environment	122
6.5	Pure Noise-Induced Transition Phenomena: A Noise-Induced Critical Point in a Model of Genic Selection	128
6.5.1	The Model	128
6.5.2	A Noise-Induced Critical Point	129
6.5.3	Critical Exponents for Noise-Induced Critical Behavior	133
6.5.4	Genic Selection in a Fluctuating Environment	136
6.6	Time-Dependent Behavior of Fokker-Planck Equations: Systems Reducible to a Linear Problem	139
6.6.1	Transformation to Linear SDE	139
6.6.2	Examples: The Verhulst Model and Hongler's Model	141
6.7	Eigenfunction Expansion of the Transition Probability Density ...	143
6.7.1	Spectral Theory of the Fokker-Planck Operator and the Sturm-Liouville Problem	143
6.7.2	Examples: The Ornstein-Uhlenbeck Process and the Verhulst Equation	148
6.8	Critical Dynamics of Noise-Induced Transitions	154
7.	Noise-Induced Transitions in Physics, Chemistry, and Biology	164
7.1	Noise-Induced Transitions in a Parametric Oscillator	164
7.2	Noise-Induced Transitions in an Open Chemical System: The Briggs-Rauscher Reaction	172
7.3	Optical Bistability	177
7.4	Noise-Induced Transitions and the Extinction Problem in Predator-Prey Systems	182
7.4.1	Two-State Predator Model	183

7.4.2 Cell-Mediated Immune Surveillance: An Example of Two-State Predator Systems	187
7.5 Illuminated Chemical Systems	189
7.5.1 Sensitivity of Biphotonic Systems to Light Intensity Fluctuations	190
7.5.2 Illuminated Photothermal Systems	194
7.5.3 Steady-State Properties for a Fluctuating Light Source	196
8. External Colored Noise	201
8.1 Modeling of Environmental Fluctuations Revisited	202
8.2 Some General Remarks on Stochastic Differential Equations with Colored Noise	204
8.3 Real External Noise: A Class of Soluble Models	206
8.4 Perturbation Expansion in the Bandwidth Parameter for the Probability Density	210
8.4.1 Verhulst Model	225
8.4.2 The Genetic Model	225
8.5 Switching-Curve Approximation	226
8.6 An Approximate Evolution Operator for Systems Coupled to Colored Noise	228
8.7 Nonlinear External Noise	235
8.7.1 Theoretical Aspects	235
8.7.2 The Freedericksz Transition in Nematic Liquid Crystals ...	240
8.7.3 Electrohydrodynamic Instabilities and External Noise	247
8.8 Turbulence and External Noise	252
9. Markovian Dichotomous Noise: An Exactly Soluble Colored-Noise Case	258
9.1 Markovian Dichotomous Noise: Formalism	258
9.2 Phase Diagrams for D Noise-Induced Transitions	271
9.2.1 The Verhulst Model	271
9.2.2 The Genetic Model	273
9.2.3 Hongler's Model	278
9.2.4 Dichotomous Periodic Forcing	280
9.3 Electrically Excitable Membranes	282
9.3.1 The Hodgkin-Huxley Axon and the Dichotomous Voltage Noise	285
9.3.2 Phase Diagrams for Sodium and Potassium Conductance of the Hodgkin and Huxley Axon	288
10. The Symbiosis of Noise and Order – Concluding Remarks	293
Appendix	295
A. Generalized Stochastic Processes	295
B. Markov Property of Solutions of Ito SDE's	298
C. The Stratonovich Calculus Obeys Classical Rules	299
D. Critical Exponents of the Mean Field Theory	300
References	303
Subject Index	315