Contents

I **Balance Laws** . . . 1
- 1.1 Formulation of the Balance Law . . . 2
- 1.2 Reduction to Field Equations . . . 2
- 1.3 Change of Coordinates and a Trace Theorem . . . 6
- 1.4 Systems of Balance Laws . . . 11
- 1.5 Companion Balance Laws . . . 12
- 1.6 Weak and Shock Fronts . . . 15
- 1.7 Survey of the Theory of *BV* Functions . . . 16
- 1.8 *BV* Solutions of Systems of Balance Laws . . . 20
- 1.9 Rapid Oscillations and the Stabilizing Effect of Companion Balance Laws . . . 22
- 1.10 Notes . . . 23

II **Introduction to Continuum Physics** . . . 25
- 2.1 Bodies and Motions . . . 25
- 2.2 Balance Laws in Continuum Physics . . . 28
- 2.3 The Balance Laws of Continuum Thermomechanics . . . 31
- 2.4 Material Frame Indifference . . . 35
- 2.5 Thermoelasticity . . . 36
- 2.6 Thermoviscoelasticity . . . 43
- 2.7 Incompressibility . . . 46
- 2.8 Relaxation . . . 47
- 2.9 Notes . . . 48

III **Hyperbolic Systems of Balance Laws** . . . 51
- 3.1 Hyperbolicity . . . 51
- 3.2 Entropy-Entropy Flux Pairs . . . 52
- 3.3 Examples of Hyperbolic Systems of Balance Laws . . . 54
- 3.4 Notes . . . 64

IV The Cauchy Problem ... 67
- 4.1 The Cauchy Problem: Classical Solutions ... 67
- 4.2 Breakdown of Classical Solutions ... 70
- 4.3 The Cauchy Problem: Weak Solutions ... 73
- 4.4 Nonuniqueness of Weak Solutions ... 74
- 4.5 Entropy Admissibility Condition ... 75
- 4.6 The Vanishing Viscosity Approach ... 78
- 4.7 Initial-Boundary-Value-Problems ... 82
- 4.8 Notes ... 85

V Entropy and the Stability of Classical Solutions ... 87
- 5.1 Convex Entropy and the Existence of Classical Solutions ... 88
- 5.2 The Role of Damping and Relaxation ... 92
- 5.3 Convex Entropy and the Stability of Classical Solutions ... 98
- 5.4 Involutions ... 101
- 5.5 Contingent Entropies and Polyconvexity ... 111
- 5.6 Notes ... 120

VI The L^1 Theory for Scalar Conservation Laws ... 123
- 6.1 The Cauchy Problem: Perseverance and Demise of Classical Solutions ... 124
- 6.2 Admissible Weak Solutions and their Stability Properties ... 126
- 6.3 The Method of Vanishing Viscosity ... 131
- 6.4 Solutions as Trajectories of a Contraction Semigroup ... 136
- 6.5 The Layering Method ... 142
- 6.6 Relaxation ... 146
- 6.7 A Kinetic Formulation ... 153
- 6.8 Fine Structure of L^∞ Solutions ... 159
- 6.9 Initial-Boundary-Value Problems ... 162
- 6.10 The L^1 Theory for Systems of Conservation Laws ... 166
- 6.11 Notes ... 170

VII Hyperbolic Systems of Balance Laws in One-Space Dimension ... 173
- 7.1 Balance Laws in One-Space Dimension ... 173
- 7.2 Hyperbolicity and Strict Hyperbolicity ... 180
- 7.3 Riemann Invariants ... 183
- 7.4 Entropy-Entropy Flux Pairs ... 188
- 7.5 Genuine Nonlinearity and Linear Degeneracy ... 190
- 7.6 Simple Waves ... 192
- 7.7 Explosion of Weak Fronts ... 197
- 7.8 Breakdown of Classical Solutions ... 198
- 7.9 Weak Solutions ... 201
- 7.10 Notes ... 202

VIII Admissible Shocks ... 205
8.1 Strong Shocks, Weak Shocks, and Shocks of Moderate Strength 205
8.2 The Hugoniot Locus ... 207
8.3 The Lax Shock Admissibility Criterion; Compressive, Overcompressive and Undercompressive Shocks . 213
8.4 The Liu Shock Admissibility Criterion ... 218
8.5 The Entropy Shock Admissibility Criterion ... 220
8.6 Viscous Shock Profiles ... 224
8.7 Nonconservative Shocks ... 234
8.8 Notes ... 235

IX Admissible Wave Fans and the Riemann Problem ... 239
9.1 Self-similar Solutions and the Riemann Problem ... 239
9.2 Wave Fan Admissibility Criteria ... 242
9.3 Solution of the Riemann Problem via Wave Fan Curves ... 243
9.4 Systems with Genuinely Nonlinear or Linearly Degenerate Characteristic Families ... 246
9.5 General Strictly Hyperbolic Systems ... 248
9.6 Failure of Existence or Uniqueness; Delta Shocks and Transitional Waves ... 253
9.7 The Entropy Rate Admissibility Criterion ... 256
9.8 Viscous Wave Fans ... 263
9.9 Interaction of Wave Fans ... 274
9.10 Breakdown of Weak Solutions ... 281
9.11 Self-similar Solutions for Multidimensional Conservation Laws ... 285
9.12 Notes ... 289

X Generalized Characteristics ... 295
10.1 BV Solutions ... 295
10.2 Generalized Characteristics ... 296
10.3 Extremal Backward Characteristics ... 298
10.4 Notes ... 300

XI Genuinely Nonlinear Scalar Conservation Laws ... 301
11.1 Admissible BV Solutions and Generalized Characteristics ... 302
11.2 The Spreading of Rarefaction Waves ... 305
11.3 Regularity of Solutions ... 306
11.4 Divides, Invariants and the Lax Formula ... 311
11.5 Decay of Solutions Induced by Entropy Dissipation ... 314
11.6 Spreading of Characteristics and Development of N-Waves ... 316
11.7 Confinement of Characteristics and Formation of Saw-toothed Profiles ... 318
11.8 Comparison Theorems and L^1 Stability ... 320
11.9 Genuinely Nonlinear Scalar Balance Laws ... 328

11.10 Balance Laws with Linear Excitation 332
11.11 An Inhomogeneous Conservation Law 335
11.12 Notes 340

XII Genuinely Nonlinear Systems of Two Conservation Laws 343
12.1 Notation and Assumptions 343
12.2 Entropy-Entropy Flux Pairs and the Hodograph Transformation 345
12.3 Local Structure of Solutions 348
12.4 Propagation of Riemann Invariants
Along Extremal Backward Characteristics 351
12.5 Bounds on Solutions 368
12.6 Spreading of Rarefaction Waves 381
12.7 Regularity of Solutions 386
12.8 Initial Data in L^1 388
12.9 Initial Data with Compact Support 392
12.10 Periodic Solutions 398
12.11 Notes 403

XIII The Random Choice Method 405
13.1 The Construction Scheme 405
13.2 Compactness and Consistency 408
13.3 Wave Interactions, Approximate Conservation Laws
and Approximate Characteristics
in Genuinely Nonlinear Systems 414
13.4 The Glimm Functional for Genuinely Nonlinear Systems 418
13.5 Bounds on the Total Variation
for Genuinely Nonlinear Systems 423
13.6 Bounds on the Supremum for Genuinely Nonlinear Systems ... 425
13.7 General Systems 428
13.8 Wave Tracing 430
13.9 Inhomogeneous Systems of Balance Laws 433
13.10 Notes 440

XIV The Front Tracking Method and Standard Riemann Semigroups .. 443
14.1 Front Tracking for Scalar Conservation Laws 444
14.2 Front Tracking for Genuinely Nonlinear
Systems of Conservation Laws 447
14.3 The Global Wave Pattern 451
14.4 Approximate Solutions 452
14.5 Bounds on the Total Variation 455
14.6 Bounds on the Combined Strength of Pseudoshocks 458
14.7 Compactness and Consistency 460
14.8 Continuous Dependence on Initial Data 462
14.9 The Standard Riemann Semigroup 466
14.10 Uniqueness of Solutions 468

14.11 Continuous Glimm Functionals, Spreading of Rarefaction Waves, and Structure of Solutions ... 473
14.12 Stability of Strong Waves ... 476
14.13 Notes ... 479

XV Construction of *BV* Solutions by the Vanishing Viscosity Method .. 483
15.1 The Main Result ... 483
15.2 Road Map to the Proof of Theorem 15.1.1 ... 485
15.3 The Effects of Diffusion ... 487
15.4 Decomposition into Viscous Traveling Waves ... 490
15.5 Transversal Wave Interactions ... 494
15.6 Interaction of Waves of the Same Family ... 498
15.7 Energy Estimates ... 503
15.8 Stability Estimates ... 506
15.9 Notes ... 508

XVI Compensated Compactness ... 511
16.1 The Young Measure ... 512
16.2 Compensated Compactness and the div-curl Lemma ... 513
16.3 Measure-Valued Solutions for Systems of Conservation Laws and Compensated Compactness ... 515
16.4 Scalar Conservation Laws ... 518
16.5 A Relaxation Scheme for Scalar Conservation Laws ... 519
16.6 Genuinely Nonlinear Systems of Two Conservation Laws ... 523
16.7 The System of Isentropic Elasticity ... 525
16.8 The System of Isentropic Gas Dynamics ... 530
16.9 Notes ... 533

Bibliography ... 537

Author Index ... 613

Subject Index ... 621