

Pei-Chu Hu
Chung-Chun Yang

Value Distribution Theory Related to Number Theory

Contents

Preface	ix
1 Heights	
1.1 Field extensions	1
1.2 Fields with valuations	11
1.2.1 Absolute values	11
1.2.2 Extensions of absolute values	16
1.3 Discriminant of field extensions	21
1.3.1 Discriminant	21
1.3.2 Dedekind domain	25
1.3.3 Different	29
1.4 Product formula	33
1.5 Hermitian geometry	36
1.5.1 Gauges of elementary operators	36
1.5.2 Hypersurfaces	45
1.5.3 Nochka weights	47
1.6 Basic geometric notions	50
1.6.1 Varieties	50
1.6.2 Function fields	52
1.6.3 Dimensions	55
1.6.4 Differential forms	57
1.6.5 Divisors	60
1.6.6 Linear systems	64
1.7 Weil functions	69
1.8 Heights in number fields	73
1.9 Functorial properties of heights	78
1.10 Gauss' lemma	83

2 Nevanlinna Theory

2.1	Notions in complex geometry	89
2.1.1	Holomorphic functions	89
2.1.2	Complex manifolds	93
2.1.3	Meromorphic mappings	103
2.1.4	Holomorphic vector bundles	106
2.1.5	Holomorphic line bundles	116
2.1.6	Hermitian manifolds	121
2.1.7	Parabolic manifolds	126
2.1.8	Inequalities on symmetric polynomials	128
2.2	Kobayashi hyperbolicity	130
2.2.1	Hyperbolicity	130
2.2.2	Measure hyperbolicity	132
2.2.3	Open problems	138
2.3	Characteristic functions	139
2.4	Growth of rational functions	146
2.5	Lemma of the logarithmic derivative	150
2.6	Second main theorem	157
2.7	Notes on the second main theorem	163
2.8	The Cartan-Nochka theorem	167
2.9	First main theorem for line bundles	175
2.10	Jacobian sections	182
2.11	Stoll's theorems	191
2.12	Carlson-Griffiths-King theory	197
2.12.1	Second main theorem for line bundles	197
2.12.2	Griffiths' and Lang's conjectures	205

3 Topics in Number Theory

3.1	Elliptic curves	213
3.1.1	The geometry of elliptic curves	213
3.1.2	Modular functions	218
3.1.3	Cusp forms	222
3.1.4	Problems in elliptic curves	226
3.1.5	Hyperelliptic and rational curves	231
3.2	The <i>abc</i> -conjecture	232
3.3	Mordell's conjecture and generalizations	237
3.4	Fermat equations and Waring's problem	240
3.5	Thue-Siegel-Roth's theorem	243
3.6	Schmidt's subspace theorem	245
3.7	Vojta's conjectures	249

3.8	Subspace theorems on hypersurfaces	256
3.8.1	Main results and problems	256
3.8.2	Equivalence of Theorems 3.65 and 3.68	260
3.8.3	Proof of Theorem 3.65	263
3.8.4	Proof of Theorem 2.63	270
3.9	Vanishing sums in function fields	272
3.9.1	Algebraic function fields	272
3.9.2	Mason's inequality	275
3.9.3	No vanishing subsums	278
4	Function Solutions of Diophantine Equations	
4.1	Nevanlinna's third main theorem	287
4.2	Generalized Mason's theorem	296
4.3	Generalized <i>abc</i> -conjecture	301
4.4	Generalized Hall's conjecture	304
4.5	Borel's theorem and its analogues	308
4.5.1	Borel's theorem	308
4.5.2	Siu-Yeung's theorem	315
4.5.3	Analogue of Borel's theorem	317
4.6	Meromorphic solutions of Fermat equations	319
4.7	Waring's problem for meromorphic functions	328
4.8	Holomorphic curves into a complex torus	336
4.8.1	Elliptic curves	337
4.8.2	Complex torus	339
4.9	Hyperbolic spaces of lower dimensions	341
4.9.1	Picard's theorem	341
4.9.2	Hyperbolic curves	343
4.9.3	Hyperbolic surfaces	350
4.9.4	Uniqueness polynomials	355
4.10	Factorization of functions	356
4.11	Wiman-Valiron theory	364
5	Functions over Non-Archimedean Fields	
5.1	Equidistribution formula	371
5.2	Second main theorem of meromorphic functions	379
5.3	Equidistribution formula for hyperplanes	384
5.4	Non-Archimedean Cartan-Nochka theorem	389
5.5	Holomorphic curves into projective varieties	394
5.5.1	Equidistribution formula for hypersurfaces	394
5.5.2	Characteristic functions for divisors	397

5.6	The <i>abc</i> -theorem for meromorphic functions	399
5.7	The <i>abc</i> -theorem for entire functions	403
5.8	Non-Archimedean Borel theorem	406
5.9	Waring's problem over function fields	410
5.10	Picard-Berkovich's theorem	413
6	Holomorphic Curves in Canonical Varieties	
6.1	Variations of the first main theorem	421
6.1.1	Green's formula	421
6.1.2	Analogue of Vojta's conjecture	423
6.2	Meromorphic connections	429
6.3	Siu theory	435
6.3.1	Siu's inequality	435
6.3.2	Generalization of Siu's theorem	441
6.4	Bloch-Green's conjecture	444
6.5	Green-Griffiths' conjecture	447
6.6	Notes on Griffiths' and Lang's conjectures	450
7	Riemann's ζ-function	
7.1	Riemann's functional equation	461
7.2	Converse theorems	467
7.3	Riemann's hypothesis	473
7.4	Hadamard's factorization	480
7.5	Nevanlinna's formula	486
7.6	Carleman's formula	493
7.7	Levin's formula	501
7.8	Notes on Nevanlinna's conjecture	504
	Bibliography	507
	Symbols	529
	Index	533