

V. D. Liseikin

A Computational Differential Geometry Approach to Grid Generation

Second Edition

With 81 Figures
Including 3 Color Figures



Springer

Contents

Part I Geometric Background to Grid Technology

1	Introductory Notions	5
1.1	Representation of Physical Geometries	5
1.2	General Concepts Related to Grids	8
1.2.1	Grid Cells	8
1.2.2	Requirements Imposed on Cells and Grids	10
1.3	Grid Generation Models	16
1.3.1	Mapping Approach	17
1.3.2	Requirements Imposed on Mathematical Models	21
1.3.3	Algebraic Methods	22
1.3.4	Differential Methods	24
1.3.5	Variational Methods	28
1.4	Comprehensive Codes	32
2	General Coordinate Systems in Domains	35
2.1	Jacobi Matrix	35
2.2	Coordinate Lines, Tangential Vectors, and Grid Cells	36
2.3	Coordinate Surfaces and Normal Vectors	38
2.4	Representation of Vectors Through the Base Vectors	40
2.5	Metric Tensors	42
2.5.1	Covariant Metric Tensor	42
2.5.2	Line Element	43
2.5.3	Contravariant Metric Tensor	44
2.5.4	Relations Between Covariant and Contravariant Elements	45
2.6	Cross Product	46
2.6.1	Geometric Meaning	47
2.6.2	Relation to Volumes	48
2.6.3	Relation to Base Vectors	49
2.7	Relations Concerning Second Derivatives	49
2.7.1	Christoffel Symbols of Domains	50
2.7.2	Differentiation of the Jacobian	52
2.7.3	Basic Identity	52

3	Geometry of Curves	55
3.1	Curves in Multidimensional Space	55
3.1.1	Definition	55
3.1.2	Basic Curve Vectors	55
3.2	Curves in Three-Dimensional Space	57
3.2.1	Basic Vectors	57
3.2.2	Curvature	58
3.2.3	Torsion	59
4	Multidimensional Geometry	61
4.1	Tangent and Normal Vectors and Tangent Plane	61
4.2	First Groundform	63
4.2.1	Covariant Metric Tensor	63
4.2.2	Contravariant Metric Tensor	65
4.3	Generalization to Riemannian Manifolds	67
4.3.1	Definition of the Manifolds	67
4.3.2	Example of a Riemannian Manifold	70
4.3.3	Christoffel Symbols of Manifolds	71
4.4	Tensors	74
4.4.1	Definition	75
4.4.2	Examples of Tensors	76
4.4.3	Tensor Operations	79
4.5	Basic Invariants	81
4.5.1	Beltrami's Differential Parameters	81
4.5.2	Measure of Relative Spacing	82
4.5.3	Measure of Relative Clustering	84
4.5.4	Mean Curvature	85
4.6	Geometry of Hypersurfaces	85
4.6.1	Normal Vector to a Hypersurface	85
4.6.2	Second Fundamental Form	90
4.6.3	Surface Curvatures	90
4.6.4	Formulas of the Mean Curvature	91
4.7	Relations to the Principal Curvatures of Two-Dimensional Surfaces	106
4.7.1	Second Fundamental Form	106
4.7.2	Principal Curvatures	107

Part II Algorithms and Applications of Advanced Grid Technology

5	Comprehensive Grid Models	117
5.1	Formulation of Differential Grid Generators	119
5.1.1	Beltramian Operator	119
5.1.2	Boundary Value Problem for Grid Equations	120

5.1.3	Interpretation as a Multidimensional Equidistribution Principle	124
5.1.4	Realization of Specified Grids	125
5.1.5	Extension to Diffusion Equations	128
5.1.6	Familiar Grid Equations.....	129
5.2	Variational Formulations	131
5.2.1	Functional of Grid Smoothness	132
5.2.2	Diffusion Functional	139
5.3	Formulation of Monitor Metrics	140
5.3.1	General Formulas for Covariant Elements.....	141
5.3.2	Formulations of Contravariant Elements.....	148
5.3.3	Specification of Individual Monitor Metrics	150
5.3.4	Monitor Metrics for Generating Balanced Grids.....	158
6	Inverted Equations	161
6.1	General Forms of Equations	161
6.1.1	Relations to Beltrami Equations.....	161
6.1.2	Resolved Grid Equations	163
6.1.3	Fluxes-Sources Equations.....	165
6.2	Equations for Classical Monitor Metrics	168
6.2.1	Domain Grid Equations for a Diagonal Monitor Metric	169
6.2.2	Domain Grid Equations with Respect to the Metric of a Monitor Surface	173
6.2.3	Surface Grid Equations for Some Special Monitor Metrics	176
6.2.4	Surface Grid Equations with Respect to the Metric of a Monitor Surface	178
6.3	Role of the Mean Curvature	182
6.3.1	Mean Curvature and Inverted Beltrami Grid Equations	182
6.3.2	Mean Curvature and Control of Grid Clustering	185
6.4	Practical Grid Equations	207
6.4.1	Equations for Generating Grids on Curves	208
6.4.2	Equations for Generating Grids on Two-Dimensional Surfaces	210
6.4.3	Equations for Generating Grids in Domains.....	214
7	Numerical Implementation of Grid Generators	219
7.1	Method of Fractional Steps	219
7.1.1	One-Dimensional Equation	219
7.1.2	Two-Dimensional Equations	222
7.1.3	Three-Dimensional Equations.....	232
7.2	Method of Minimization of Energy Functional.....	236
7.2.1	Generation of Fixed Grids	236
7.2.2	Adaptive Grid Generation	242
7.2.3	Numerical Examples	255

7.3	Generation of Multi-Block Grids	255
7.3.1	Block-Structured Grids	257
7.4	Application of Layer-Type Functions to Grid Codes	267
7.4.1	Specification of Basic Functions	267
7.4.2	Numerical Grids Aligned to Vector-Fields	268
7.4.3	Application to Grid Clustering	273
7.4.4	Application to Formulation of Weight Functions for Generating Balanced Grids	275
References		279
Index		289