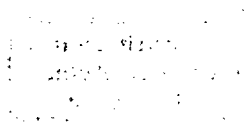


Jürgen Jost

Postmodern Analysis

Third Edition

With 14 Figures



 Springer

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