

Contents

1	Kinetic theory	1
1.1	The Boltzmann equation	1
1.2	Collision transformations	2
1.3	Collision kernels	8
1.4	Boundary conditions	11
1.5	Physical properties of gas flows	12
1.5.1	Physical quantities and units	12
1.5.2	Macroscopic flow properties	13
1.5.3	Molecular flow properties	14
1.5.4	Measurements	15
1.5.5	Air at standard conditions	15
1.6	Properties of the collision integral	16
1.7	Moment equations	21
1.8	Criterion of local equilibrium	24
1.9	Scaling transformations	28
1.10	Comments and bibliographic remarks	30
2	Related Markov processes	33
2.1	Boltzmann type piecewise-deterministic Markov processes	33
2.1.1	Free flow and state space	33
2.1.2	Construction of sample paths	34
2.1.3	Jump behavior	35
2.1.4	Extended generator	39
2.2	Heuristic derivation of the limiting equation	41
2.2.1	Equation for measures	41
2.2.2	Equation for densities	43
2.2.3	Boundary conditions	46
2.3	Special cases and bibliographic remarks	47
2.3.1	Boltzmann equation and boundary conditions	47
2.3.2	Boltzmann type processes	48
2.3.3	History	55

3	Stochastic weighted particle method	65
3.1	The DSMC framework	67
3.1.1	Generating the initial state	67
3.1.2	Decoupling of free flow and collisions	68
3.1.3	Limiting equations	69
3.1.4	Calculation of functionals	69
3.2	Free flow part	71
3.2.1	Modeling of boundary conditions	71
3.2.2	Modeling of inflow	74
3.3	Collision part	78
3.3.1	Cell structure	78
3.3.2	Fictitious collisions	80
3.3.3	Majorant condition	81
3.3.4	Global upper bound for the relative velocity norm	83
3.3.5	Shells in the velocity space	85
3.3.6	Temperature time counter	89
3.4	Controlling the number of particles	97
3.4.1	Collision processes with reduction	97
3.4.2	Convergence theorem	99
3.4.3	Proof of the convergence theorem	103
3.4.4	Construction of reduction measures	119
3.5	Comments and bibliographic remarks	132
3.5.1	Some Monte Carlo history	132
3.5.2	Time counting procedures	132
3.5.3	Convergence and variance reduction	134
3.5.4	Nanbu's method	136
3.5.5	Approximation order	140
3.5.6	Further references	145
4	Numerical experiments	147
4.1	Maxwellian initial state	149
4.1.1	Uniform approximation of the velocity space	150
4.1.2	Stability of moments	150
4.1.3	Tail functionals	151
4.1.4	Hard sphere model	156
4.2	Relaxation of a mixture of two Maxwellians	157
4.2.1	Convergence of DSMC	159
4.2.2	Convergence of SWPM	161
4.2.3	Tail functionals	164
4.2.4	Hard sphere model	169
4.3	BKW solution of the Boltzmann equation	171
4.3.1	Convergence of moments	172
4.3.2	Tail functionals	175
4.4	Eternal solution of the Boltzmann equation	178
4.4.1	Power functionals	180

4.4.2	Tail functionals	181
4.5	A spatially one-dimensional example	181
4.5.1	Properties of the shock wave problem	182
4.5.2	Mott-Smith model	185
4.5.3	DSMC calculations	189
4.5.4	Comparison with the Mott-Smith model	191
4.5.5	Histograms	194
4.5.6	Bibliographic remarks	197
4.6	A spatially two-dimensional example	198
4.6.1	Explicit formulas in the collisionless case	200
4.6.2	Case with collisions	206
4.6.3	Influence of a hot wall	207
4.6.4	Bibliographic remarks	210
A	Auxiliary results	211
A.1	Properties of the Maxwell distribution	211
A.2	Exact relaxation of moments	213
A.3	Properties of the BKW solution	217
A.4	Convergence of random measures	220
A.5	Existence of solutions	223
B	Modeling of distributions	229
B.1	General techniques	229
B.1.1	Acceptance-rejection method	229
B.1.2	Transformation method	230
B.1.3	Composition method	231
B.2	Uniform distribution on the unit sphere	231
B.3	Directed distribution on the unit sphere	232
B.4	Maxwell distribution	234
B.5	Directed half-space Maxwell distribution	236
B.6	Initial distribution of the BKW solution	239
B.7	Initial distribution of the eternal solution	241
	References	243
	Index	255