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Mechanics

From Newton's Laws
to Deterministic Chaos

Fourth Edition

With 174 Figures and 117 Problems with Complete Solutions



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