

A.V. Skorokhod

Basic Principles and Applications of Probability Theory

Edited by Yu.V. Prokhorov

Translated by B. D. Seckler

 Springer

Contents

I. Probability. Basic Notions. Structure. Methods	1
II. Markov Processes and Probability Applications in Analysis	143
III. Applied Probability	191
Author Index	275
Subject Index	277

Probability. Basic Notions. Structure. Methods

Contents

1	Introduction	5
1.1	The Nature of Randomness	5
1.1.1	Determinism and Chaos	6
1.1.2	Unpredictability and Randomness	6
1.1.3	Sources of Randomness.	7
1.1.4	The Role of Chance	8
1.2	Formalization of Randomness	9
1.2.1	Selection from Among Several Possibilities. Experiments. Events	9
1.2.2	Relative Frequencies. Probability as an Ideal Relative Frequency	12
1.2.3	The Definition of Probability	13
1.3	Problems of Probability Theory	14
1.3.1	Probability and Measure Theory	15
1.3.2	Independence	15
1.3.3	Asymptotic Behavior of Stochastic Systems	16
1.3.4	Stochastic Analysis	17
2	Probability Space	19
2.1	Finite Probability Space	19
2.1.1	Combinatorial Analysis	19
2.1.2	Conditional Probability	21
2.1.3	Bernoulli's Scheme. Limit Theorems	24
2.2	Definition of Probability Space	27
2.2.1	σ -algebras. Probability	27
2.2.2	Random Variables. Expectation	29
2.2.3	Conditional Expectation	31
2.2.4	Regular Conditional Distributions	34
2.2.5	Spaces of Random Variables. Convergence	35
2.3	Random Mappings	38
2.3.1	Random Elements	38

2.3.2	Random Functions	42
2.3.3	Random Elements in Linear Spaces	44
2.4	Construction of Probability Spaces	46
2.4.1	Finite-dimensional Space	46
2.4.2	Function Spaces	47
2.4.3	Linear Topological Spaces. Weak Distributions	50
2.4.4	The Minlos-Sazonov Theorem	51
3	Independence	53
3.1	Independence of σ -Algebras	53
3.1.1	Independent Algebras	53
3.1.2	Conditions for the Independence of σ -Algebras	55
3.1.3	Infinite Sequences of Independent σ -Algebras	56
3.1.4	Independent Random Variables	57
3.2	Sequences of Independent Random Variables	59
3.2.1	Sums of Independent Random Variables	59
3.2.2	Kolmogorov's Inequality	61
3.2.3	Convergence of Series of Independent Random Variables	63
3.2.4	The Strong Law of Large Numbers	65
3.3	Random Walks	67
3.3.1	The Renewal Scheme	67
3.3.2	Recurrency	71
3.3.3	Ladder Functionals	74
3.4	Processes with Independent Increments	78
3.4.1	Definition	78
3.4.2	Stochastically Continuous Processes	80
3.4.3	Lévy's Formula	83
3.5	Product Measures	86
3.5.1	Definition	86
3.5.2	Absolute Continuity and Singularity of Measures	87
3.5.3	Kakutani's Theorem	88
3.5.4	Absolute Continuity of Gaussian Product Measures ...	91
4	General Theory of Stochastic Processes and Random Functions	93
4.1	Regular Modifications	93
4.1.1	Separable Random Functions	94
4.1.2	Continuous Stochastic Processes	96
4.1.3	Processes With at Most Jump Discontinuities	97
4.1.4	Markov Processes	98
4.2	Measurability	100
4.2.1	Existence of a Measurable Modification	100
4.2.2	Mean-Square Integration	101
4.2.3	Expansion of a Random Function in an Orthogonal Series	103

4.3	Adapted Processes	104
4.3.1	Stopping Times	105
4.3.2	Progressive Measurability	106
4.3.3	Completely Measurable and Predictable σ -Algebras . . .	107
4.3.4	Completely Measurable and Predictable Processes . . .	108
4.4	Martingales	110
4.4.1	Definition and Simplest Properties	110
4.4.2	Inequalities. Existence of the Limit	111
4.4.3	Continuous Parameter	114
4.5	Stochastic Integrals and Integral Representations of Random Functions	115
4.5.1	Random Measures	115
4.5.2	Karhunen's Theorem	116
4.5.3	Spectral Representation of Some Random Functions . .	117
5	Limit Theorems	119
5.1	Weak Convergence of Distributions	119
5.1.1	Weak Convergence of Measures in Metric Spaces	119
5.1.2	Weak Compactness	122
5.1.3	Weak Convergence of Measures in R^d	123
5.2	Ergodic Theorems	124
5.2.1	Measure-Preserving Transformations	124
5.2.2	Birkhoff's Theorem	126
5.2.3	Metric Transitivity	130
5.3	Central Limit Theorem and Invariance Principle	132
5.3.1	Identically Distributed Terms	132
5.3.2	Lindeberg's Theorem	133
5.3.3	Donsker-Prokhorov Theorem	135
	Historic and Bibliographic Comments	139
	References	141