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# Signal Extraction

Efficient Estimation, 'Unit Root'-Tests  
and Early Detection of Turning Points

With 80 Figures and 15 Tables

 Springer

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## Part I Theory

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