

Contents

Introduction	xi
0 A Motivating Problem (K.)	1
0.1 Dimensions ≤ 4	1
0.2 Dimension 6	2
0.3 Dimension 5	4
1 Introduction to the Novikov and the Borel Conjecture (L.)	5
1.1 The Original Formulation of the Novikov Conjecture	5
1.2 Invariance Properties of the L -Class	6
1.3 The Borel Conjecture	8
2 Normal Bordism Groups (K.)	11
2.1 Normal Bordism Groups	11
2.2 Rational Computation of Normal Bordism Groups	12
2.3 Rational Computation of Oriented Bordism Groups	14
3 The Signature (K.)	17
3.1 The Definition of the Signature	18
3.2 The Bordism Invariance of the Signature	18
3.3 Multiplicativity and other Properties of the Signature	20
3.4 Geometric Interpretation of Cohomology and the Intersection Form	21
4 The Signature Theorem and the Novikov Conjecture (K.)	25
4.1 The Signature Theorem	25
4.2 Higher Signatures	27
4.3 The Novikov Conjecture	28
4.4 The Pontrjagin Classes are not Homeomorphism Invariants	29
5 The Projective Class Group and the Whitehead Group (L.)	33
5.1 The Projective Class Group	33
5.2 The First Algebraic K -Group	35

5.3	The Whitehead Group	39
5.4	The Bass-Heller-Swan Decomposition	40
6	Whitehead Torsion (L.)	43
6.1	Whitehead Torsion of a Chain Map	43
6.2	The Cellular Chain Complex of the Universal Covering	47
6.3	The Whitehead Torsion of a Cellular Map	49
6.4	Simple Homotopy Equivalences	52
7	The Statement and Consequences of the s-Cobordism Theorem (L.)	55
8	Sketch of the Proof of the s-Cobordism Theorem (L.)	59
8.1	Handlebody Decompositions	59
8.2	Handlebody Decompositions and CW -Structures	61
8.3	Reducing the Handlebody Decomposition	63
8.4	Handlebody Decompositions and Whitehead Torsion	65
9	From the Novikov Conjecture to Surgery (K.)	69
9.1	The Structure Set	69
9.2	The Assembly Idea	70
10	Surgery Below the Middle Dimension I: An Example (K.)	75
10.1	Surgery and its Trace	75
10.2	The Effect on the Fundamental Group and Homology Groups	76
10.3	Application to Knottings	77
11	Surgery Below the Middle Dimension II: Systematically (K.)	79
11.1	The Effect of Surgery in Homology and Homotopy	79
11.2	Surgery below the Middle Dimension	81
11.3	Construction of Certain 6-Manifolds	84
12	Surgery in the Middle Dimension I (K.)	87
12.1	Motivation for the Surgery Obstruction Groups	87
12.2	Unimodular Hermitian Forms	88
12.3	The L -Groups in Dimensions $4m$	89
12.4	The L -Groups in Other Dimensions	90
13	Surgery in the Middle Dimension II (K.)	93
13.1	Equivariant Intersection Numbers	93
13.2	Stably Free Modules	94
13.3	The Quadratic Refinement	95
13.4	The Surgery Obstruction	97

14	Surgery in the Middle Dimension III (K.)	99
14.1	Stable Diffeomorphism Classification	99
14.2	The Surgery Obstruction is a Bordism Invariant	101
14.3	The Main Result	101
14.4	Proof of the Main Theorem	103
14.5	The Exact Surgery Sequence	106
14.6	Stable Classification of Certain 6-Manifolds	107
15	An Assembly Map (K.)	109
15.1	More on the Definition of the Assembly Map	109
15.2	The Surgery Version of the Novikov Conjecture	112
16	The Novikov Conjecture for $\mathbb{Z}^n$ (K.)	113
16.1	The Idea of the Proof	113
16.2	Reduction to Mapping Tori	113
16.3	The Proof for Rank 1	115
16.4	The Generalization to Higher Rank	117
17	Poincaré Duality and Algebraic L-Groups (L. and Varisco)	119
17.1	Poincaré duality	119
17.2	Algebraic L -groups	124
18	Spectra (L.)	133
18.1	Basic Notions about Spectra	133
18.2	Homotopy Pushouts and Homotopy Pullbacks for Spaces	135
18.3	Homotopy Pushouts and Homotopy Pullbacks for Spectra	138
18.4	(Co-)Homology Theories Associated to Spectra	139
18.5	K -Theory and L -Theory Spectra	141
18.6	The Chern Character for Homology Theories	143
18.7	The Bordism Group Associated to a Vector Bundle	144
18.8	The Thom Space of a Vector Bundle	145
18.9	The Pontrjagin–Thom Construction	145
18.10	The Stable Version of the Pontrjagin–Thom Construction	146
18.11	The Oriented Bordism Ring	148
18.12	Stable Homotopy	149
18.13	The Thom Isomorphism	150
18.14	The Rationalized Oriented Bordism Ring	150
18.15	The Integral Oriented Bordism Ring	151
19	Classifying Spaces of Families (L.)	153
19.1	Basics about G - CW -Complexes	153
19.2	The Classifying Space for a Family	156

19.3	Special Models	157
19.3.1	The Family of All Subgroups and the Trivial Family	157
19.3.2	Operator Theoretic Model	157
19.3.3	Discrete Subgroups of Almost Connected Lie Groups	157
19.3.4	Simply Connected Non-Positively Curved Manifolds	158
19.3.5	CAT(0)-Spaces	158
19.3.6	Trees with Finite Isotropy Groups	158
19.3.7	Amalgamated Products and HNN-Extensions	158
19.3.8	Arithmetic Groups	159
19.3.9	Outer Automorphism Groups of Free groups	160
19.3.10	Mapping Class groups	160
19.3.11	One-Relator Groups	160
19.3.12	Special Linear Groups of (2,2)-Matrices	161
20	Equivariant Homology Theories and the Meta-Conjecture (L.)	163
20.1	The Meta-Conjecture	163
20.2	Formulation of the Farrell- Jones and the Baum- Connes Conjecture	164
20.3	Equivariant Homology Theories	165
20.4	The Construction of Equivariant Homology Theories from Spectra	168
21	The Farrell-Jones Conjecture (L.)	173
21.1	The Bass-Heller-Swan Decomposition in Arbitrary Dimensions . .	173
21.2	Decorations in L -Theory and the Shaneson Splitting	174
21.3	Changing the Family	176
21.4	The Farrell-Jones Conjecture for Torsionfree Groups	177
21.5	The Farrell-Jones Conjecture and the Borel Conjecture	180
21.6	The Passage from $\mathcal{FIN}$ to $\mathcal{WC}$	181
22	The Baum-Connes Conjecture (L.)	185
22.1	Index Theoretic Interpretation of the Baum-Connes Assembly Map	185
22.2	The Baum-Connes Conjecture for Torsionfree Groups	186
22.3	The Trace Conjecture in the Torsionfree Case	187
22.4	The Kadison Conjecture	187
22.5	The Stable Gromov-Lawson-Rosenberg Conjecture	188
22.6	The Choice of the Family $\mathcal{FIN}$ in the Baum-Connes Conjecture .	189
23	Relating the Novikov, the Farrell-Jones and the Baum-Connes Conjectures (L.)	191
23.1	The Farrell-Jones Conjecture and the Novikov Conjecture	191
23.2	Relating Topological K -Theory and L -Theory	195
23.3	The Baum-Connes Conjecture and the Novikov Conjecture	197

24	Miscellaneous (L.)	201
24.1	Status of the Conjectures	201
24.2	Methods of Proof	206
24.3	Computations for Finite Groups	206
24.3.1	Topological K -Theory for Finite Groups	206
24.3.2	Algebraic K -Theory for Finite Groups	207
24.3.3	Algebraic L -Theory for Finite Groups	208
24.4	Rational Computations	208
24.4.1	Rationalized Topological K -Theory for Infinite Groups . . .	209
24.4.2	Rationalized Algebraic K -Theory for Infinite Groups	210
24.4.3	Rationalized Algebraic L -Theory for Infinite Groups	211
24.5	Integral Computations	211
25	Exercises	215
26	Hints to the Solutions of the Exercises	223
	References	236
	Index	254
	Notation	261
	Schedules	263
	List of Participants	267