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SYMMETRY RULES

How Science and Nature Are Founded on Symmetry

With 86 Figures and 4 Tables

Springer

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