

Contents

0	Introduction	1
1	Lattice theory	9
1.1	Basic properties	9
1.2	Classification of indefinite unimodular lattices	16
1.3	Embeddings of lattices	20
2	Reflection groups and their fundamental domains	27
2.1	Reflection groups and fundamental domains	27
2.2	Reflection groups associated with lattices	32
3	Complex analytic surfaces	35
3.1	Basics of complex analytic surfaces	35
3.2	Classification of complex analytic surfaces	41
3.3	Elliptic surfaces and their singular fibers	44
4	$K3$ surfaces and examples	55
4.1	Definition and examples of $K3$ surfaces	55
4.2	Reflection group associated with non-singular rational curves and the Kähler cone	60
4.3	Kummer surfaces	62
4.4	The Kummer surface associated with a curve of genus 2	65
4.5	Torelli theorem for 2-dimensional complex tori	68
5	Bounded symmetric domains of type IV and deformations of complex structures	75
5.1	Bounded symmetric domains of type IV	75
5.2	Deformations of complex structures and the Kodaira–Spencer map	80
6	The Torelli-type theorem for $K3$ surfaces	87
6.1	Periods of $K3$ surfaces and the Torelli-type theorem	87

6.2	Local isomorphism of the period map (local Torelli theorem)	91
6.3	The Torelli-type theorem for Kummer surfaces	94
6.4	Density of the periods of Kummer surfaces	103
6.5	Behavior of the Kähler cones under a deformation	109
6.6	Proof of the Torelli-type theorem for $K3$ surfaces	112
7	Surjectivity of the period map of $K3$ surfaces	119
7.1	The period map of marked Kähler $K3$ surfaces	119
7.2	Surjectivity of the period map of $K3$ surfaces	120
7.3	Outline of a proof of the surjectivity of the period map of projective $K3$ surfaces	122
8	Application of the Torelli-type theorem to automorphisms	127
8.1	Automorphism group of a projective $K3$ surface	127
8.2	Action of the automorphism group on the transcendental lattice . .	128
8.3	A finite group that can be realized as an automorphism group of a $K3$ surface	133
8.4	Automorphisms of $K3$ surfaces of order 2	134
9	Enriques surfaces	137
9.1	Periods of Enriques surfaces	137
9.2	Non-singular rational curves and elliptic curves on Enriques surfaces	147
9.3	Automorphism groups of Enriques surfaces	152
9.4	Examples of Enriques surfaces	155
10	Application to the moduli space of plane quartic curves	171
10.1	Plane quartics and del Pezzo surfaces of degree 2	171
10.2	$K3$ surfaces associated with plane quartics	180
10.3	The moduli space of plane quartics and a complex ball	184
11	Finite groups of symplectic automorphisms of $K3$ surfaces and the Mathieu group	191
11.1	Niemeier lattices and the Mathieu group	191
11.2	Finite symplectic automorphisms and the Mathieu group	195
12	Automorphism group of the Kummer surface associated with a curve of genus 2	201
12.1	The Leech lattice and the even unimodular lattice of signature $(1, 25)$	201
12.2	Néron–Severi lattice of the Kummer surface	204
12.3	Classical automorphisms of the Kummer surface	207

12.4 The Kummer surface and the Leech roots	209
12.5 Automorphism group of a generic Kummer surface	215
Bibliography	223
Index	231