

Contents

Contents of Volume 1 — IX

Preface to the second edition — XV

1	Free Euclidean quantum field and Ornstein–Uhlenbeck processes — 1
1.1	Background — 1
1.2	Boson Fock space — 3
1.2.1	Second quantization — 3
1.2.2	The case $\mathscr{H} = L^2(\mathbb{R}^d)$ — 9
1.2.3	The case $\mathscr{H} = \mathbb{C}1$ — 12
1.2.4	Segal–Bargmann space — 14
1.2.5	Segal fields — 16
1.2.6	Wick product — 20
1.2.7	Exponentials of creation and annihilation operators — 21
1.2.8	The case $\mathscr{H} = L^2(\mathbb{R}^d)$ — 27
1.3	$\mathscr{Q}$ -spaces — 43
1.3.1	Gaussian random processes — 43
1.3.2	Wiener–Itô–Segal isomorphism and positivity improving — 46
1.3.3	Hypercontractivity — 52
1.3.4	Lorentz covariant quantum fields — 56
1.4	Existence of $\mathscr{Q}$ -spaces — 57
1.4.1	Countable product spaces — 57
1.4.2	Bochner theorem and Minlos theorem — 59
1.5	Functional integral representation of the Euclidean quantum field — 66
1.5.1	Basic results in Euclidean quantum field theory — 66
1.5.2	Markov property of projections — 75
1.5.3	Feynman–Kac–Nelson formula — 78
1.5.4	van Hove Hamiltonian — 80
1.6	Infinite dimensional Ornstein–Uhlenbeck processes — 82
1.6.1	Abstract theory of Gaussian measures on Hilbert spaces — 82
1.6.2	Abstract theory of Borel measures on Hilbert spaces — 90
1.6.3	Fock space as a function space — 97
1.6.4	Infinite dimensional Ornstein–Uhlenbeck process — 100
1.6.5	Markov property — 106
1.6.6	Regular conditional Gaussian probability measures — 108
1.6.7	Feynman–Kac–Nelson formula by path measures — 110
1.7	Connection with infinite dimensional stochastic analysis and Malliavin calculus — 111
1.7.1	Finite dimensional case — 111

1.7.2	Stochastic derivative and Cameron–Martin space —	115
1.7.3	Malliavin derivative and divergence operator on $L^2(\mathcal{X})$ —	118
1.7.4	Wiener–Itô chaos expansion —	121
1.7.5	Malliavin derivative and divergence operator on Wiener chaos —	124
1.7.6	Infinite dimensional Ornstein–Uhlenbeck semigroup —	126
1.7.7	Malliavin derivative on white noise space —	127
2	The Nelson model by path measures —	131
2.1	Preliminaries —	131
2.2	The Nelson model in Fock space —	132
2.2.1	Definition of the Nelson model —	132
2.2.2	Infrared and ultraviolet divergences —	135
2.2.3	Embedded eigenvalues —	136
2.3	The Nelson model in function space —	137
2.3.1	Infinite dimensional Ornstein–Uhlenbeck processes and $P(\phi)_1$ -processes —	137
2.3.2	Euclidean field and Brownian motion —	143
2.3.3	Extension to general external potential —	148
2.4	Nelson model with Kato-class potential —	151
2.5	Existence and uniqueness of the ground state —	155
2.5.1	Uniqueness —	155
2.5.2	Existence —	157
2.6	Ground state expectations —	164
2.6.1	General expressions —	164
2.6.2	Ground state expectations for second quantized operators —	170
2.6.3	Ground state expectation for fractional powers of the number operator —	176
2.6.4	Ground state expectations of field operators —	180
2.6.5	Gaussian domination —	182
2.7	Infrared divergence —	184
2.8	Gibbs measure associated with the ground state —	189
2.8.1	Local convergence and Gibbs measures —	189
2.8.2	$P(\phi)_1$ -process associated with the Nelson Hamiltonian —	196
2.8.3	Applications to ground state expectations —	199
2.9	Carmona-type estimates —	207
2.9.1	Exponential decay of bound states: upper bound —	207
2.9.2	Exponential decay of bound states: lower bound —	208
2.10	Martingale properties and applications —	211
2.10.1	Martingale properties —	211
2.10.2	Exponential decay of bound states —	213
2.11	Ultraviolet divergence —	215
2.11.1	Energy renormalization —	215

2.11.2	Regularized interaction — 219
2.11.3	Removal of ultraviolet cutoff on Fock vacuum — 232
2.11.4	Uniform lower bound and removal of ultraviolet cutoff — 237
2.11.5	Functional integral representation of the ultraviolet renormalized Nelson model — 239
2.11.6	Gibbs measures and applications — 253
2.11.7	Weak coupling limit and removal of ultraviolet cutoff — 262
2.12	Translation invariant Nelson model — 267
2.12.1	Definition of translation invariant Nelson model — 267
2.12.2	Functional integral representation — 270
2.12.3	Existence of ground state — 273
2.12.4	Gibbs measure associated with the ground state of Nelson model with zero total momentum — 275
2.12.5	$P(\phi)_1$ -process associated with Nelson Hamiltonian with zero total momentum — 277
2.12.6	Removal of ultraviolet cutoff — 280
2.12.7	Ground state energy and ultraviolet renormalization term — 285
2.12.8	Gibbs measures and applications — 288
2.13	Polaron model — 291
2.13.1	Definition of the polaron model — 291
2.13.2	Functional integral representation — 292
2.13.3	Removal of ultraviolet cutoff — 294
2.14	Functional central limit theorem — 296
2.14.1	Gibbs measures with no external potential — 296
2.14.2	Diffusive behavior — 306
2.14.3	Diffusion matrix and effective mass — 314
3	The Pauli–Fierz model by path measures — 317
3.1	Preliminaries — 317
3.1.1	Introduction — 317
3.1.2	Lagrangian QED — 318
3.1.3	Classical variant of nonrelativistic QED — 322
3.2	The Pauli–Fierz model in nonrelativistic QED — 324
3.2.1	The Pauli–Fierz model in Fock space — 324
3.2.2	The Pauli–Fierz model in function space — 329
3.2.3	Markov property — 334
3.3	Functional integral representation for the Pauli–Fierz Hamiltonian — 337
3.3.1	Hilbert space-valued stochastic integrals — 337
3.3.2	Functional integral representation — 341
3.3.3	Extension to general external potential — 349
3.4	The Pauli–Fierz model with Kato-class potential — 351

3.5	Applications of functional integral representations —	355
3.5.1	Self-adjointness of the Pauli–Fierz Hamiltonian —	355
3.5.2	Positivity improving and uniqueness of the ground state —	364
3.5.3	Spatial decay of bound states —	370
3.6	Path measure associated with the ground state of Pauli–Fierz Hamiltonian —	371
3.6.1	Path measure with double stochastic integrals —	371
3.6.2	Expression in terms of iterated stochastic integrals —	375
3.6.3	Weak convergence and Gibbs measures —	378
3.6.4	Gaussian domination of ground states —	381
3.7	Translation invariant Pauli–Fierz model —	382
3.8	The Pauli–Fierz model with spin —	389
3.8.1	Counting measures —	389
3.8.2	Review of classical cases —	390
3.8.3	Definition of the Pauli–Fierz Hamiltonian with spin —	392
3.8.4	Symmetry and polarization —	395
3.8.5	Scalar representations —	401
3.8.6	Fock representations —	405
3.8.7	Preparation of functional integral representations —	407
3.8.8	Functional integral representations —	421
3.8.9	Translation invariant Pauli–Fierz Hamiltonian with spin —	432
3.9	Relativistic Pauli–Fierz model —	439
3.9.1	Definition of relativistic Pauli–Fierz Hamiltonian —	439
3.9.2	Functional integral representations —	442
3.9.3	Self-adjointness —	447
3.9.4	Nonrelativistic limit of relativistic Pauli–Fierz Hamiltonian —	457
3.9.5	Relativistic Pauli–Fierz model with relativistic Kato-class potential —	460
3.9.6	Martingale properties —	465
3.9.7	Spatial decay of bound states —	469
3.9.8	Gaussian domination of ground states —	471
3.9.9	Path measure associated with the ground state of relativistic Pauli–Fierz Hamiltonian —	473
3.9.10	Translation invariant relativistic Pauli–Fierz model —	474
3.9.11	Nonrelativistic limit of translation invariant relativistic Pauli–Fierz Hamiltonian —	478
4	Spin-boson model by path measures —	479
4.1	Definitions —	479
4.2	Functional integral representation for the spin-boson Hamiltonian —	483
4.2.1	Preliminaries —	483

- 4.2.2 Spin process — 484
- 4.2.3 Functional integral representation — 486
- 4.3 Existence and uniqueness of ground state — 488
- 4.4 Gibbs measure associated with the ground state — 490
 - 4.4.1 Local convergence and Gibbs measures — 490
 - 4.4.2 Ground state properties — 491
 - 4.4.3 van Hove representation — 499
- 4.5 Rabi Hamiltonian — 501

- 5 Notes and references — 505
 - Notes to Chapter 1 — 505
 - Notes to Chapter 2 — 507
 - Notes to Chapter 3 — 514
 - Notes to Chapter 4 — 519

- Bibliography — 521

- Index — 533