

Contents

Preface — IX

1 Governing equations — 1

- 1.1 Introduction — 1
- 1.2 The Navier–Stokes equations for incompressible viscous flow — 1
 - 1.2.1 Model — 1
 - 1.2.2 Variational formulations of the Navier–Stokes equations — 4
- 1.3 Appendix A: Derivation of the flow models — 7
 - 1.3.1 Remark on the momentum equation (1.1) — 7
 - 1.3.2 Remark on the variational formulation (1.17) — 8
 - 1.3.3 Remark on the variational formulation (1.20) — 10

2 Operator splitting methods for initial value problems: Application to the Navier–Stokes equations — 13

- 2.1 A family of initial value problems — 13
- 2.2 Operator splitting of the autonomous initial value problem (2.1) by the Lie scheme — 14
- 2.3 Operator splitting of the autonomous initial value problem (2.1) by the Strang scheme — 15
- 2.4 Fractional-step schemes à la Marchuk–Yanenko — 16
- 2.5 Application to the Navier–Stokes equations — 18
- 2.6 Appendix B: Remarks on the Lie scheme (2.3) — 20

3 Advection problems — 23

- 3.1 The wave-like equation method — 23
- 3.2 Properties of scheme (3.17)–(3.19) — 28

4 Numerical solution of the generalized Stokes-type subproblems — 37

- 4.1 Mathematical properties of the generalized Stokes problem (4.1) — 37
- 4.2 The Stokes operator — 39
- 4.3 Existence results for the generalized Stokes problem (4.6) — 44
- 4.4 A saddle-point interpretation of the generalized Stokes problem (4.1) — 45
- 4.5 A gradient method for the generalized Stokes problem — 47
- 4.6 Conjugate gradient algorithms for the generalized Stokes problems — 60
- 4.7 Appendix C: Conjugate gradient methods for the solution of minimization problems in Hilbert spaces — 70
 - 4.7.1 Conjugate gradient solution of linear variational problems in Hilbert spaces — 70

5	Finite-element approximation of the Navier–Stokes equations — 79
5.1	Finite-element methods for the Stokes problem — 80
5.1.1	Some observations — 80
5.1.2	Discrete spaces — 84
5.1.3	Approximation of the boundary conditions — 85
5.1.4	Formulation of the discrete generalized Stokes problem — 86
5.1.5	On the convergence of the finite element approximations of the generalized Stokes problem — 89
5.2	Finite-element implementation of the projection/wave-like equation methods — 101
5.2.1	Projection methods — 101
5.3	On the numerical solution of the discrete subproblems — 103
5.4	Appendix D: Proofs of Lemma 5.4.1, Theorems 5.1.5, 5.1.8, and Corollary 5.1.6 — 107
6	Numerical experiments: Description and results — 117
6.1	Performance of the preconditioned conjugate gradient method and numerical errors for the Bercovier–Pironneau finite-element approximation of the Stokes problem — 117
6.2	A two-dimensional wall-driven cavity problem — 121
6.3	A three-dimensional wall-driven cavity problem — 132
6.3.1	Numerical results — 133
7	Further applications (I): A distributed Lagrange multiplier/fictitious-domain method for simulating lid-driven viscous flows in a hemispherical cavity — 144
7.1	Introduction to fictitious-domain methods: Principle, historical facts and synopsis — 144
7.2	Governing equations — 146
7.3	A distributed Lagrange multiplier/fictitious-domain formulation — 147
7.4	Finite-element approximation of problem (7.17)–(7.20) — 148
7.5	Time discretization of problem (7.23)–(7.27) by operator-splitting methods — 150
7.5.1	Solution of the subproblems in schemes (7.28)–(7.32) and (7.33)–(7.36) — 152
7.6	Numerical results — 154
8	Further applications (II): On the simulation of the lid-driven cavity flow of an Oldroyd-B fluid — 164
8.1	Introduction — 164
8.2	Formulation of the problem — 165
8.3	Scheme and discretizations — 166

8.4	Numerical experiments —	171
8.4.1	Regular triangular meshes —	172
8.4.2	Locally-refined triangular meshes —	175
9	Further applications (III): A distributed Lagrange multiplier/fictitious-domain method for simulating particles settling in an Oldroyd-B fluid —	180
9.1	Introduction —	180
9.2	Mathematical formulation —	181
9.2.1	The governing equations —	181
9.2.2	A fictitious-domain formulation —	183
9.3	Numerical methods and an operator-splitting scheme —	186
9.3.1	Finite-element approximation —	186
9.3.2	An operator-splitting scheme —	188
9.4	On the solutions of the fractional step subproblems —	191
9.4.1	Solution of the discrete degenerated Stokes subproblem (9.47) —	192
9.4.2	Solution of the rigid body motion enforcement problems (9.55) —	194
9.5	Numerical results —	196
9.6	Conclusions —	201

Bibliography — 203

Index — 213