

Contents

Guide for the Reader	xvii
List of Symbols	xxiii
Chapter IX. The Hilbert Scheme	1
1. Introduction	1
2. The idea of the Hilbert scheme	4
3. Flatness	12
4. Construction of the Hilbert scheme	19
5. The characteristic system	27
6. Mumford's example	40
7. Variants of the Hilbert scheme	43
8. Tangent space computations	49
9. C^n families of projective manifolds	56
10. Bibliographical notes and further reading	64
11. Exercises	65
Chapter X. Nodal curves	79
1. Introduction	79
2. Elementary theory of nodal curves	83
3. Stable curves	99
4. Stable reduction	104
5. Isomorphisms of families of stable curves	113
6. The stable model, contraction, and projection	117
7. Clutching	126
8. Stabilization	127
9. Vanishing cycles and the Picard–Lefschetz transformation ...	143
10. Bibliographical notes and further reading	161
11. Exercises	161
Chapter XI. Elementary deformation theory and some applications	167
1. Introduction	167
2. Deformations of manifolds	172
3. Deformations of nodal curves	178

4.	The concept of Kuranishi family	187
5.	The Hilbert scheme of ν -canonical curves	193
6.	Construction of Kuranishi families	203
7.	The Kuranishi family and continuous deformations	212
8.	The period map and the local Torelli theorem	216
9.	Curvature of the Hodge bundles	224
10.	Deformations of symmetric products	242
11.	Bibliographical notes and further reading	248
Chapter XII. The moduli space of stable curves		249
1.	Introduction	249
2.	Construction of moduli space as an analytic space	257
3.	Moduli spaces as algebraic spaces	268
4.	The moduli space of curves as an orbifold	274
5.	The moduli space of curves as a stack, I	279
6.	The classical theory of descent for quasi-coherent sheaves	288
7.	The moduli space of curves as a stack, II	294
8.	Deligne–Mumford stacks	299
9.	Back to algebraic spaces	307
10.	The universal curve, projections and clutchings	309
11.	Bibliographical notes and further reading	323
12.	Exercises	323
Chapter XIII. Line bundles on moduli		329
1.	Introduction	329
2.	Line bundles on the moduli stack of stable curves	332
3.	The tangent bundle to moduli and related constructions	344
4.	The determinant of the cohomology and some applications ...	347
5.	The Deligne pairing	366
6.	The Picard group of moduli space	379
7.	Mumford’s formula	382
8.	The Picard group of the hyperelliptic locus	387
9.	Bibliographical notes and further reading	396
Chapter XIV. Projectivity of the moduli space of stable curves		399
1.	Introduction	399
2.	A little invariant theory	400
3.	The invariant-theoretic stability of linearly stable smooth curves	406
4.	Numerical inequalities for families of stable curves	414
5.	Projectivity of moduli spaces	425
6.	Bibliographical notes and further reading	437

Chapter XV. The Teichmüller point of view	441
1. Introduction	441
2. Teichmüller space and the mapping class group	445
3. A little surface topology	453
4. Quadratic differentials and Teichmüller deformations	461
5. The geometry associated to a quadratic differential	472
6. The proof of Teichmüller's uniqueness theorem	479
7. Simple connectedness of the moduli stack of stable curves	483
8. Going to the boundary of Teichmüller space	485
9. Bibliographical notes and further reading	497
10. Exercises	498
Chapter XVI. Smooth Galois covers of moduli spaces	501
1. Introduction	501
2. Level structures on smooth curves	508
3. Automorphisms of stable curves	515
4. Compactifying moduli of curves with level structure; a first attempt	518
5. Admissible G -covers	525
6. Automorphisms of admissible covers	536
7. Smooth covers of \overline{M}_g	544
8. Totally unimodular lattices	551
9. Smooth covers of $\overline{M}_{g,n}$	556
10. Bibliographical notes and further reading	562
11. Exercises	562
Chapter XVII. Cycles in the moduli spaces of stable curves	565
1. Introduction	565
2. Algebraic cycles on quotients by finite groups	566
3. Tautological classes on moduli spaces of curves	570
4. Tautological relations and the tautological ring	573
5. Mumford's relations for the Hodge classes	585
6. Further considerations on cycles on moduli spaces	596
7. The Chow ring of $\overline{M}_{0,P}$	599
8. Bibliographical notes and further reading	604
9. Exercises	605
Chapter XVIII. Cellular decomposition of moduli spaces	609
1. Introduction	609
2. The arc system complex	613
3. Ribbon graphs	616
4. The idea behind the cellular decomposition of $M_{g,n}$	623
5. Uniformization	624
6. Hyperbolic geometry	627

7.	The hyperbolic spine and the definition of Ψ	636
8.	The equivariant cellular decomposition of Teichmüller space ..	643
9.	Stable ribbon graphs	648
10.	Extending the cellular decomposition to a partial compactification of Teichmüller space	652
11.	The continuity of $\hat{\Psi}$	655
12.	Odds and ends	661
13.	Bibliographical notes and further reading	665
 Chapter XIX. First consequences of the cellular decomposition		667
1.	Introduction	667
2.	The vanishing theorems for the rational homology of $M_{g,P}$..	670
3.	Comparing the cohomology of $\overline{M}_{g,n}$ to the one of its boundary strata	673
4.	The second rational cohomology group of $\overline{M}_{g,n}$	676
5.	A quick overview of the stable rational cohomology of $M_{g,n}$ and the computation of $H^1(M_{g,n})$ and $H^2(M_{g,n})$	683
6.	A closer look at the orbicell decomposition of moduli spaces ..	690
7.	Combinatorial expression for the classes ψ_i	694
8.	A volume computation	699
9.	Bibliographical notes and further reading	708
10.	Exercises	709
 Chapter XX. Intersection theory of tautological classes		717
1.	Introduction	717
2.	Witten's generating series	721
3.	Virasoro operators and the KdV hierarchy	726
4.	The combinatorial identity	729
5.	Feynman diagrams and matrix models	734
6.	Kontsevich's matrix model and the equation $L_2Z = 0$	745
7.	A nonvanishing theorem	750
8.	A brief review of equivariant cohomology and the virtual Euler–Poincaré characteristic	754
9.	The virtual Euler–Poincaré characteristic of $M_{g,n}$	759
10.	A very quick tour of Gromov–Witten invariants	766
11.	Bibliographical notes and further reading	771
12.	Exercises	773
 Chapter XXI. Brill–Noether theory on a moving curve		779
1.	Introduction	779
2.	The relative Picard variety	781
3.	Brill–Noether varieties on moving curves	788
4.	Looijenga's vanishing theorem	796

5.	The Zariski tangent spaces to the Brill–Noether varieties	802
6.	The μ_1 homomorphism	808
7.	Lazarsfeld’s proof of Petri’s conjecture	814
8.	The normal bundle and Horikawa’s theory	819
9.	Ramification	835
10.	Plane curves	845
11.	The Hurwitz scheme and its irreducibility	854
12.	Plane curves and g_d^1 ’s	863
13.	Unirationality results	872
14.	Bibliographical notes and further reading	879
15.	Exercises	885
Bibliography		903
Index		945