

Contents

1	Introduction	9
2	The Classical ANOVA Decomposition	21
2.1	Weighted Lebesgue Spaces and Orthonormal Systems .	22
2.1.1	Smoothness and Weighted Decay	26
2.1.2	Examples for Complete Orthonormal Systems .	30
2.1.3	Approximation with Partial Sums	35
2.2	Projection, ANOVA Terms and the ANOVA Decomposition	39
2.3	Effective Dimensions and the Truncated ANOVA Decomposition	51
3	Fast Multiplication with Grouped Transformations	65
3.1	Non-Equispaced Transformations	67
3.2	Grouped Transformations	71
3.2.1	Grouped Index Sets	71
3.2.2	Fast Multiplication	75
3.2.3	Implementation	91

4	High-Dimensional Explainable ANOVA Approximation	95
4.1	Least-Squares Approximation with Regularization . . .	96
4.1.1	The Overdetermined Case	100
4.1.2	The Underdetermined Case and Smoothness Information	103
4.2	Explainable Method, Attribute Ranking and Active Set Detection	106
4.2.1	Attribute Ranking and Active Set Detection . . .	113
4.3	Approximation Errors	116
4.3.1	Worst-Case Error	116
4.3.2	Individual Error	128
5	Numerical Experiments with Synthetic Data	139
5.1	Periodic B-Spline Function	141
5.2	Non-Periodic B-Spline Function	150
5.3	Friedman Functions	155
5.3.1	Friedman 1	156
5.3.2	Friedman 2	161
5.3.3	Friedman 3	163
5.3.4	Runtime	166
6	Numerical Experiments with Real Data	167
6.1	Forest Fires	171
6.2	Energy Efficiency	175
6.3	Airfoil Self-Noise	176
6.4	California Housing	178
6.5	Ailerons	179
7	Conclusion	183
	Bibliography	189