

Contents

1	Introduction	9
2	Preliminaries and notations	17
3	Fourier approximation on the torus	21
3.1	Fourier analysis on the torus	21
3.2	Fourier approximation with rank-1 lattices	23
3.3	Approximation error bounds on the torus	24
3.4	Algorithms on the torus	25
3.4.1	Efficient evaluation and reconstruction of trigonometric polynomials . .	25
3.4.2	Multiple rank-1 lattices	27
3.4.3	Sparse fast Fourier transform	27
3.5	Discrete approximation errors	28
3.5.1	The ℓ_∞ -approximation error	28
3.5.2	The ℓ_2 -approximation error	29
4	Torus-to-$\mathbb{R}^d$ transformation mappings	31
4.1	Torus-to- $\mathbb{R}^d$ transformations	32
4.2	Exemplary transformations	33
4.3	Weighted Hilbert spaces on $\mathbb{R}^d$	34
4.4	Smoothness properties of transformed functions	36
4.5	Approximation of transformed functions	42
4.5.1	L_2 -approximation error	44
4.5.2	L_∞ -approximation error	45
4.6	Fast algorithms and discrete approximation errors on $\mathbb{R}^d$	46
4.7	Numerics for the error function transformation	48
4.7.1	Single rank-1 lattices in dimension $d = 1$	51
4.7.2	Single rank-1 lattices in dimension $d = 2$	52
4.7.3	Multiple rank-1 lattices in dimension $d = 4$	53
4.7.4	Suitable frequency sets in up to dimension $d = 7$	54
4.8	Summary of the numerics on $\mathbb{R}^d$	55
5	Torus-to-cube transformation mappings	59
5.1	Torus-to-cube transformations	61
5.2	Exemplary transformations	62
5.3	Weighted Hilbert spaces on the cube	63
5.4	Smoothness properties of transformed functions	65
5.5	Approximation of transformed functions	70

5.5.1	L_∞ -approximation error	71
5.5.2	L_2 -approximation error	73
5.6	Fast algorithms and discrete approximation errors on the cube	74
5.7	Half-periodic cosine and Chebyshev approximation	76
5.7.1	Cosine approximation with tent-transformed sampling nodes	77
5.7.2	Chebyshev approximation	79
5.7.3	Comparison of the orthonormal systems	82
5.8	Numerics on the cube	83
5.8.1	Approximation of a first-order B-spline in dimensions $d \in \{1, 2, 4, 7\}$	84
5.8.2	Approximation of a second-order B-spline in dimensions $d \in \{1, 2, 4, 7\}$	88
5.8.3	Sparse frequency sets	90
5.9	Summary of the numerics on the cube	92
6	Conclusion	95
	Alphabetical Index	97