

Contents

Preface	v
1 A non-existence result for a generalized radial Brezis–Nirenberg problem	1
by <i>Rafael D. Benguria and Soledad Benguria</i>	
1 Introduction	1
2 Non-existence of solutions, via the Pohozaev virial identity, using the conformal Laplacian	5
3 Non-existence of solutions using the Rellich–Pohozaev argument and a “Hardy-type” inequality	9
4 An illustrative example	12
Bibliography	14
2 Friedrichs-type inequalities in arbitrary domains	17
by <i>Andrea Cianchi and Vladimir Maz'ya</i>	
1 Introduction	17
2 Background and notations	20
3 First-order inequalities	23
4 Second-order inequalities	27
5 Inequalities for the symmetric gradient	33
6 Sketches of proofs	34
Bibliography	37
3 Ari Laptev and the Journal of Spectral Theory	39
by <i>E. Brian Davies</i>	
4 Critical magnetic field for 2D magnetic Dirac–Coulomb operators and Hardy inequalities	41
by <i>Jean Dolbeault, Maria J. Esteban and Michael Loss</i>	
1 Introduction and main results	41
2 Homogeneous Hardy-like inequalities	49
3 The minimization problem	50
4 The 2D magnetic Dirac–Coulomb operator with an Aharonov–Bohm magnetic field	53
5 Non-relativistic limit	56
A Appendix	57
Bibliography	60

5	The Feshbach–Schur map and perturbation theory	65
	<i>by Geneviève Dussan, Israel Michael Sigal and Benjamin Stamm</i>	
1	Set-up and result	65
2	Perturbation estimates	70
3	Application: The ground state energy of the Helium-type ions	76
A	The eigenfunctions of the Hydrogen-like Hamiltonian	84
B	The numerical approximation of the constants $w_1, w_2, w_1^{\text{as}}$ and w_2^{as}	84
	Bibliography	87
6	Adiabatic and non-adiabatic evolution of wave packets and applications to initial value representations	89
	<i>by Clotilde Fermanian Kammerer, Caroline Lasser and Didier Robert</i>	
1	Introduction	89
2	Propagation of Gaussian states and the Herman–Kluk approximation for scalar equations	92
3	Herman–Kluk formula in the adiabatic setting	94
4	What about smooth crossings?	98
5	A sketchy proof for Herman–Kluk approximations	102
	Bibliography	111
7	Length scales for BEC in the dilute Bose gas	115
	<i>by Søren Fournais</i>	
1	Introduction	115
2	Energy in small boxes	119
3	Localization to small boxes	127
A	Facts about the scattering solution	132
	Bibliography	132
8	The periodic Lieb–Thirring inequality	135
	<i>by Rupert L. Frank, David Gontier and Mathieu Lewin</i>	
1	The periodic Lieb–Thirring inequality	135
2	The one-dimensional integrable case $\gamma = \frac{3}{2}$	140
3	Numerical simulations in 1D and 2D	143
A	A minimization problem with entropy	146
B	Computation of the density of states of V_k	147
	Bibliography	153
9	Semiclassical asymptotics for a class of singular Schrödinger operators	155
	<i>by Rupert L. Frank and Simon Larson</i>	
1	Introduction	155
2	Preliminaries	157
3	Local asymptotics	161

4	From local to global asymptotics	170
A	Properties of P_ν	172
	Bibliography	175
10	On the spectral properties of the Bloch–Torrey equation in infinite periodically perforated domains	177
	<i>by Denis S. Grebenkov, Bernard Helffer and Nicolas Moutal</i>	
1	Introduction	177
2	The Bloch–Torrey operator in the perforated whole plane	178
3	Floquet approach for y -periodic problems	179
4	The Bloch–Torrey operator in a perforated cylinder continued	184
5	Quasi-modes and non-emptiness of the spectrum	190
6	Conclusion	192
A	Generalized Lax–Milgram theorem	193
B	Spectrum and Weyl’s sequences	194
	Bibliography	195
11	Counting bound states with maximal Fourier multipliers	197
	<i>by Dirk Hundertmark, Peer Kunstmann, Tobias Ried and Semjon Vugalter</i>	
1	Introduction	197
2	The splitting trick	201
	Bibliography	206
12	Sharp dimension estimates of the attractor of the damped 2D Euler–Bardina equations	209
	<i>by Alexei Ilyin and Sergey Zelik</i>	
1	Introduction	209
2	Global attractor	212
3	Dimension estimate	214
4	A sharp lower bound	216
5	Appendix	222
	Bibliography	227
13	Upper estimates for the electronic density in heavy atoms and molecules	231
	<i>by Victor Ivrii</i>	
1	Introduction	231
2	Main intermediate inequality	233
3	Estimates of the averaged electronic density	236
4	Estimates of the correlation function	238
5	Proof of Theorem 1.1	242
	Bibliography	245

14 Non-linear Schrödinger equation in a uniform magnetic field	247
<i>by Thomas F. Kieffer and Michael Loss</i>	
1 Introduction	247
2 Non-linear magnetic Schrödinger equation	249
3 The non-linear Pauli equation	260
Bibliography	264
15 Large k behavior of d-bar problems for domains with a smooth boundary	267
<i>by Christian Klein, Johannes Sjöstrand and Nikola Stoilov</i>	
1 Introduction	267
2 Main result	269
Bibliography	275
16 Heat kernel estimates for two-dimensional relativistic Hamiltonians with magnetic field	277
<i>by Hynek Kovařík</i>	
1 Introduction	277
2 Radial magnetic field	278
3 Aharonov–Bohm-type magnetic fields	283
Bibliography	288
17 A version of Watson lemma for Laplace integrals and some applications	289
<i>by Stanislas Kupin and Sergey Naboko[†]</i>	
1 Introduction	289
2 Proofs of the results	293
3 Some corollaries	297
Bibliography	300
18 Wehrl-type coherent state entropy inequalities for $SU(1, 1)$ and its $AX + B$ subgroup	301
<i>by Elliott H. Lieb and Jan Philip Solovej</i>	
1 Introduction	301
2 Unitary representations and coherent states for the $AX + B$ group . .	304
3 Generalized conjecture for the $AX + B$ group and a partial result . .	305
4 An analytic formulation	308
5 Some unitary $SU(1, 1)$ representations and their coherent states . . .	310
6 The $SU(1, 1)$ quantum channels	312
Bibliography	312

19 Blow-ups for the Horn–Kapranov parametrization of the classical discriminant	315
<i>by Evgeny Mikhalkin, Vitaly Stepanenko and Avgust Tsikh</i>	
1 Introduction	315
2 Extremal discriminant and factorization of its truncations	318
3 Parametrizations of zero sets of discriminants	322
4 Blow-ups of Horn–Kapranov parametrizations and proof of the theorem	324
Bibliography	328
20 Eigenvalue estimates and asymptotics for weighted pseudodifferential operators with singular measures in the critical case	331
<i>by Grigori Rozenblum and Eugene Shargorodsky</i>	
1 Introduction	331
2 Setting and main results	333
3 Some reductions	337
4 Geometry considerations	341
5 Estimates	344
6 Approximation of the weight	350
7 The asymptotic formula	351
Bibliography	353
21 Relations between two parts of the spectrum of a Schrödinger operator and other remarks on the absolute continuity of the spectrum in a typical case	355
<i>by Oleg Safronov</i>	
1 Introduction	355
2 Proof of Theorem 3.4	363
Bibliography	364
22 Bogoliubov theory for many-body quantum systems	367
<i>by Benjamin Schlein</i>	
1 Introduction	367
2 Bose gases, energy and dynamics	368
3 The correlation energy of a Fermi gas	377
4 Dynamics of a Fröhlich polaron	382
Bibliography	386
23 A statistical theory of heavy atoms: Asymptotic behavior of the energy and stability of matter	389
<i>by Heinz Siedentop</i>	
1 Introduction	389
2 Bounds on the energy	392

3	Stability of matter	397
	Bibliography	401
24	Homogenization of the higher-order Schrödinger-type equations with periodic coefficients	405
	<i>by Tatiana Suslina</i>	
1	Introduction	405
2	Abstract operator-theoretic scheme	408
3	Periodic differential operators in $L_2(\mathbb{R}^d; \mathbb{C}^n)$	412
4	Application of the abstract results to $A(\mathbf{k})$	415
5	Approximation for the operator exponential of A	421
6	Homogenization of the Schrödinger-type equation	422
	Bibliography	425
25	Trace formulas for the modified Mathieu equation	427
	<i>by Leon A. Takhtajan</i>	
1	Introduction	427
2	General case	430
3	Examples	434
4	Existence of the solution $\psi_1(x, \lambda)$	440
	Bibliography	442
26	Eigenvalue accumulation and bounds for non-selfadjoint matrix differential operators related to NLS	445
	<i>by Christiane Tretter</i>	
1	Introduction	445
2	Auxiliary spectral bounds of independent interest	447
3	Invariant subspaces and non-real spectrum	450
	Bibliography	455
27	Scattering theory for Laguerre operators	457
	<i>by Dimitri R. Yafaev</i>	
1	Introduction	457
2	Jacobi operators and orthogonal polynomials	460
3	Wave operators	462
4	Time evolution	468
	Bibliography	478
	List of contributors	479