

Contents

Preface — VII

Introduction — IX

1	Introduction of Nevanlinna theory and its difference versions — 1
1.1	Basic notations and facts in the Nevanlinna theory — 1
1.2	Difference Nevanlinna theory — 11
1.2.1	Elementary difference considerations in the complex domain — 11
1.2.2	Difference variants of the logarithmic derivative lemma for meromorphic functions — 12
1.2.3	Key notions and results in difference Nevanlinna theory — 16
2	Value distribution of complex delay-differential polynomials — 21
2.1	Value distribution of difference polynomials — 21
2.2	Value distribution of delay-differential polynomials — 24
2.3	Borel exceptional values of delay-differential polynomials — 51
2.4	The zeros of $[P(f)f(z+c)]^{(k)} - \alpha(z)$ — 56
2.5	Zeros on delay-differential polynomials of exponential type — 59
3	Uniqueness of delay-differential polynomials — 67
3.1	Value sharing on f with its linear delay-differential operator — 67
3.2	Uniqueness of delay-differential polynomials — 78
4	Difference Wiman–Valiron theory — 87
4.1	Differential setting of the Wiman–Valiron theory — 87
4.2	Difference setting of the Wiman–Valiron theory — 88
4.3	Wiman–Valiron theorems for q -differences — 90
5	The linear complex delay-differential equations — 93
5.1	First-order delay-differential equations — 93
5.2	Higher-order linear complex delay-differential equations — 100
5.2.1	Higher-order homogeneous linear complex delay-differential equations — 100
5.2.2	Higher-order nonhomogeneous linear complex delay-differential equations — 106
6	Fermat-type delay-differential equations — 117
6.1	Fermat equations — 117
6.2	Fermat-type functional and differential equations — 117
6.3	Fermat difference equations — 122

6.3.1	Entire solutions of Fermat difference equations —	122
6.3.2	Meromorphic solutions of Fermat difference equations —	128
6.3.3	Fermat delay-differential equations —	132
7	Delay-differential Riccati equations —	149
7.1	Complex differential and difference Riccati equations —	149
7.2	Complex delay-differential Riccati equations —	154
8	Malmquist-type delay-differential equations —	163
8.1	Malmquist-type difference equations —	163
8.2	Malmquist-type delay-differential equations —	164
9	Nonlinear complex delay-differential equations —	195
9.1	Background —	195
9.2	Nonlinear complex delay-differential equations —	195
10	Complex q-delay-differential equations —	213
10.1	Value distribution for q -delay-differential polynomials —	213
10.2	Uniqueness on q -delay-differential polynomials —	221
10.3	q -delay-differential equations —	226
11	Systems of complex delay-differential equations —	241
11.1	Background —	241
11.2	Malmquist-type delay-differential systems —	241
11.3	Fermat-type delay-differential systems —	245
11.4	Systems of q -delay-differential equations —	251
12	Periodicity of entire functions with delay-differential polynomials —	255
12.1	Basic results on periodic functions —	255
12.2	Periodicity of differential, difference, and delay-differential polynomials —	258

Bibliography — 279

Index — 289