

Contents

Preface — VII

Notation — IX

1 Introduction and modeling — 1

1.1 Motivation — 1

1.2 Open domain with solid particles — 4

1.2.1 The cases with n -dimensional particles G_0 — 4

1.2.2 The case of $(n - 1)$ -dimensional particles contained in $\partial\Omega$ — 9

1.3 Homogenized problem: effective reaction-diffusion behavior — 10

1.3.1 Solid particles over the whole domain — 10

1.3.2 Particles over a manifold — 12

1.3.3 Particles over the boundary — 13

1.4 Different homogenization techniques — 14

1.4.1 The multiple-scales method — 14

1.4.2 The Γ -convergence method — 15

1.4.3 The two-scale convergence method — 16

1.4.4 Tartar's method of oscillating test functions — 17

1.4.5 The periodical unfolding method — 17

1.5 Structure of the proofs and main ideas: oscillating test functions — 18

1.5.1 Showing the solutions u_ε have a limit — 18

1.5.2 Characterizing an effective equation — 19

1.5.3 Study of the critical case: the appearance of the strange term — 21

1.6 A literature review — 23

1.6.1 Particles over the whole domain — 24

1.6.2 Particles over a manifold — 25

1.6.3 Particles over the boundary — 26

1.7 Novelties — 26

2 Preliminary results and comments — 29

2.1 Maximal monotone graphs. *A common roof* — 29

2.2 Variational formulation of the problems — 32

2.2.1 Formulation as variational inequalities — 33

2.2.2 Existence and uniqueness of solutions — 35

2.3 The critical scaling of the reaction constant β — 36

2.4 Uniform approximation results — 37

2.5 The range $\beta(\varepsilon) \neq \beta^*(\varepsilon)$ — 38

2.6 Comments — 39

2.6.1	Uniformly continuous σ — 39
2.6.2	Non-monotone σ — 40
3	Estimates over one periodicity cell — 43
3.1	Case of n -dimensional particles — 43
3.1.1	Extension operators — 44
3.1.2	Uniform Poincaré inequality on Ω_ε — 46
3.1.3	Sharp trace estimates on $a_\varepsilon \partial G_0$ in $\varepsilon Y \setminus a_\varepsilon \overline{G_0}$ — 46
3.1.4	Auxiliary oscillating functions in the subcritical range $a_\varepsilon^* \ll a_\varepsilon \lesssim \varepsilon$ — 49
3.1.5	Auxiliary functions in the critical case $a_\varepsilon \sim a_\varepsilon^*$: capacity problems — 52
3.2	Case of $(n - 1)$ -dimensional particles — 63
3.2.1	Trace estimates on $a_\varepsilon G_0$ in εY^+ — 63
3.2.2	Auxiliary functions in the subcritical case — 63
3.2.3	Auxiliary functions in the critical case — 65
4	Particles over the whole domain — 69
4.1	On the existence of a critical scale — 69
4.2	Integrals over S_ε — 71
4.2.1	Improved global trace inequality — 71
4.2.2	Limit of the integral over S_ε of convergent sequences in the subcritical case — 72
4.2.3	Limit of the integral over S_ε of oscillating sequences — 75
4.3	A priori estimates for u_ε — 76
4.4	Big particles $a_\varepsilon = \varepsilon$ — 77
4.4.1	The linear case $p = 2$ — 77
4.4.2	The nonlinear case $p \neq 2$ — 84
4.5	Subcritical cases $a_\varepsilon^* \ll a_\varepsilon \ll \varepsilon$ and $p > 1$ — 84
4.6	Supercritical case $a_\varepsilon \ll a_\varepsilon^*$ and $p \in (1, n)$ — 87
4.7	Critical case $a_\varepsilon \sim a_\varepsilon^*$: the anomalous (or strange) nonlinear term — 89
4.7.1	Case of G_0 a ball, $p \in (1, n)$ and $g^\varepsilon = g$ — 89
4.7.2	Case of G_0 a ball, $p = n$ and $g^\varepsilon = g$ — 101
4.7.3	Case of G_0 not a ball, $p = 2 < n$ — 102
4.7.4	Case of G_0 not a ball, $1 < p \leq n$ and $g^\varepsilon \neq 0$ — 106
4.8	The case $\beta \neq \beta^*$ — 106
4.9	Further comments — 108
4.9.1	L^1 data — 108
4.9.2	Additional properties of the strange term — 114
4.9.3	Homogenization of the effectiveness factor — 117
4.9.4	Pointwise comparison of critical and non-critical problems — 118
4.9.5	Homogenization and optimal control — 120
4.9.6	Convergence of spectra — 121

5	Particles over an interior manifold — 123
5.1	Jumps over an interface — 124
5.2	Existence of a critical scale — 125
5.3	Integrals over S_ε — 125
5.3.1	Trace theorem — 125
5.3.2	Limit of integrals over S_ε — 127
5.4	A priori estimates for u_ε — 127
5.5	Subcritical particles $a_\varepsilon^* \ll a_\varepsilon \leq \varepsilon$ — 128
5.6	Supercritical particles $a_\varepsilon \ll a_\varepsilon^*$ — 128
5.7	Critical-size particles $a_\varepsilon \sim a_\varepsilon^*$ — 129
5.8	Some remarks — 131
6	Particles over a part of the boundary — 133
6.1	Existence of a critical scale — 133
6.2	Integrals over S_ε — 133
6.3	A priori estimates — 134
6.4	Subcritical particles $a_\varepsilon^* \ll a_\varepsilon \leq \varepsilon$ — 134
6.5	Supercritical particles with $1 < p < n$ — 135
6.6	Critical-size particles — 135
6.6.1	Case of $p = 2 < n$ when G_0 is not a ball — 135
6.6.2	Case of $p = n$ when $G_0 = B_1^0$ — 136
6.7	Further comments — 140
A	Comments on the parabolic case — 141
A.1	A weak formulation in terms of a variational inequality — 142
A.2	Small subcritical particles $a_\varepsilon^* \ll a_\varepsilon \ll \varepsilon$ — 144
A.3	Supercritical particles $a_\varepsilon \ll a_\varepsilon^*$ and $p \in (1, n)$ — 144
A.4	Critical-size particles $a_\varepsilon \sim a_\varepsilon^*$ — 144
A.5	A remark on controllability — 145
B	Dynamic boundary condition — 147
B.1	Small subcritical particles $a_\varepsilon^* \ll a_\varepsilon \ll \varepsilon$ — 148
B.2	Supercritical particles $a_\varepsilon \ll a_\varepsilon^*$ — 148
B.3	Critical particles $a_\varepsilon^* \sim a_\varepsilon$ and $G_0 = B_1$ — 149
B.4	Critical particles $a_\varepsilon^* \sim a_\varepsilon$ when $p = 2$ and general G_0 — 152
C	Critical-size particles with a stochastic perturbation — 157
C.1	Some comments on the ergodicity hypothesis — 159
C.2	Convergence results — 159
C.3	Auxiliary test function — 160
C.4	Structure of the proof — 161
C.5	Final comments — 161

Bibliography — 165

Index — 181