

Contents

Part I

Maïtine Bergounioux

Second-order decomposition model for image processing: numerical experimentation — 5

- 1.1 Introduction — 5
- 1.2 Presentation of the model — 6
- 1.3 Numerical aspects — 8
 - 1.3.1 Discretized problem and algorithm — 8
 - 1.3.2 Examples — 10
 - 1.3.3 Initialization process — 12
 - 1.3.4 Convergence — 20
 - 1.3.5 Sensitivity with respect to sampling and quantification — 25
 - 1.3.6 Sensitivity with respect to parameters — 27
- 1.4 Conclusion — 31

Laurent Hoeltgen, Markus Mainberger, Sebastian Hoffmann, Joachim Weickert, Ching Hoo Tang, Simon Setzer, Daniel Johannsen, Frank Neumann, and Benjamin Doerr

Optimizing spatial and tonal data for PDE-based inpainting — 35

- 2.1 Introduction — 36
- 2.2 A review of PDE-based image compression — 38
 - 2.2.1 Data optimization — 38
 - 2.2.2 Finding good inpainting operators — 39
 - 2.2.3 Storing the data — 40
 - 2.2.4 Feature-based methods — 40
 - 2.2.5 Fast algorithms and real-time aspects — 41
 - 2.2.6 Hybrid image compression methods — 42
 - 2.2.7 Modifications, extensions and applications — 43
 - 2.2.8 Relations to other methods — 44
- 2.3 Inpainting with homogeneous diffusion — 44
- 2.4 Optimization strategies in 1D — 46
 - 2.4.1 Optimal knots for interpolating convex functions — 46
 - 2.4.2 Optimal knots for approximating convex functions — 51
- 2.5 Optimization strategies in 2D — 55
 - 2.5.1 Optimizing spatial data — 55
 - 2.5.2 Optimizing tonal data — 62
- 2.6 Extensions to other inpainting operators — 70

2.6.1	Optimizing spatial data —	71
2.6.2	Optimizing tonal data —	73
2.7	Summary and conclusions —	74

Qian Xie and Anuj Srivastava

Image registration using phase–amplitude separation — 84

3.1	Introduction —	84
3.1.1	Current literature —	85
3.1.2	Our approach —	86
3.2	Definition of phase–amplitude components —	87
3.2.1	q -Map and amplitude distance —	89
3.2.2	Relative phase and image registration —	91
3.3	Properties of registration framework —	92
3.4	Gradient method for optimization over Γ —	95
3.4.1	Basis on $T_{\gamma_{\text{id}}}(\Gamma)$ —	96
3.4.2	Mean image and group-wise registration —	97
3.5	Experiments —	97
3.5.1	Pairwise image registration —	97
3.5.2	Registering multiple images —	102
3.5.3	Image classification —	104
3.6	Conclusion —	105

Martin Eller and Massimo Fornasier

Rotation invariance in exemplar-based image inpainting — 108

4.1	Introduction to inpainting —	108
4.1.1	The inpainting problem —	108
4.1.2	Aims of this work —	112
4.1.3	Notation —	115
4.2	Rotation invariant image pattern recognition —	115
4.2.1	Patch error functions —	116
4.2.2	Circular harmonics basis —	118
4.2.3	Mutual angle detection algorithms —	121
4.2.4	Rotation invariant L^2 -error using the circular harmonics basis —	126
4.2.5	Rotation invariant gradient-based L^2 -errors and the CH-basis —	128
4.3	Rotation invariant exemplar-based inpainting —	131
4.3.1	Patch non-local means —	132
4.3.2	Patch non-local Poisson —	142
4.3.3	Numerical experiments —	147
4.4	Discussion and analysis —	154
4.4.1	Proof of convergence —	155
4.4.2	Analysis of $E_{\nabla, \Gamma}$ —	171
4.4.3	Conclusion and future perspectives —	178

José A. Iglesias and Clemens Kirisits

Convective regularization for optical flow — 184

- 5.1 Introduction — 184
- 5.2 Model — 187
 - 5.2.1 Convective acceleration — 187
 - 5.2.2 Convective regularization — 190
 - 5.2.3 Data term and contrast invariance — 193
- 5.3 Numerical solution — 195
- 5.4 Experiments — 196
- 5.5 Conclusion — 197

Elena Beretta, Monika Muszkieta, Wolf Naetar, and Otmar Scherzer

A variational method for quantitative photoacoustic tomography with piecewise constant coefficients — 202

- 6.1 Quantitative photoacoustic tomography — 202
 - 6.1.1 Introduction — 202
 - 6.1.2 Contributions of this article — 205
- 6.2 Recovery of piecewise constant coefficients — 205
- 6.3 A Mumford–Shah-like functional for qPAT — 207
 - 6.3.1 Existence of minimizers — 209
 - 6.3.2 Approximation — 211
 - 6.3.3 Minimization — 212
- 6.4 Implementation and numerical results — 215
- A Special functions of bounded variation and the SBV-compactness theorem — 220

Martin Burger, Hendrik Dirks, and Lena Frerking

On optical flow models for variational motion estimation — 225

- 7.1 Introduction — 225
- 7.2 Models — 226
 - 7.2.1 Variational models with gradient regularization — 227
 - 7.2.2 Extension of the regularizer — 228
 - 7.2.3 Bregman iterations — 230
- 7.3 Analysis — 231
 - 7.3.1 Existence of minimizers — 231
 - 7.3.2 Quantitative estimates — 234
- 7.4 Numerical solution — 236
 - 7.4.1 Primal–dual algorithm — 236
 - 7.4.2 Discretization and parameters — 239
- 7.5 Results — 241
 - 7.5.1 Error measures for velocity fields — 241
 - 7.5.2 Evaluation — 244

7.6	Conclusion and outlook —	244
7.6.1	Mass preservation —	247
7.6.2	Higher dimensions —	247
7.6.3	Joint models —	248
7.6.4	Large displacements —	248

Luca Calatroni, Chung Cao, Juan Carlos De los Reyes, Carola-Bibiane Schönlieb, and Tuomo Valkonen

Bilevel approaches for learning of variational imaging models — 252

8.1	Overview of learning in variational imaging —	252
8.2	The learning model and its analysis in function space —	257
8.2.1	The abstract model —	257
8.2.2	Existence and structure: L^2 -squared cost and fidelity —	258
8.2.3	Optimality conditions —	260
8.3	Numerical optimization of the learning problem —	262
8.3.1	Adjoint-based methods —	262
8.3.2	Dynamic sampling —	265
8.4	Learning the image model —	267
8.4.1	Total variation-type regularization —	267
8.4.2	Optimal parameter choice for TV-type regularization —	269
8.5	Learning the data model —	274
8.5.1	Variational noise models —	275
8.5.2	Single noise estimation —	276
8.5.3	Multiple noise estimation —	281
8.6	Conclusion and outlook —	283

Part II

Piernicola Bettiol and Nathalie Khalil

Non-degenerate forms of the generalized Euler–Lagrange condition for state-constrained optimal control problems — 295

- 9.1 Introduction — 295
- 9.2 Main result — 299
- 9.3 Proof of Theorem 2.1 — 301
- 9.4 Proof of Lemma 3.2 — 309
- 9.5 Example — 311

Piernicola Bettiol, Bernard Bonnard, Laetitia Giralaldi, Pierre Martinon, and Jérémy Rouot

The Purcell three-link swimmer: some geometric and numerical aspects related to periodic optimal controls — 314

- 10.1 Introduction — 315
- 10.2 First- and second-order optimality conditions — 316
- 10.3 The Purcell three-link swimmer — 318
 - 10.3.1 Mathematical model — 318
- 10.4 Local analysis for the three-link Purcell swimmer — 320
 - 10.4.1 Computations of the nilpotent approximation — 321
 - 10.4.2 Integration of extremal trajectories — 322
- 10.5 Numerical results — 330
 - 10.5.1 Nilpotent approximation — 331
 - 10.5.2 True mechanical system — 332
 - 10.5.3 The Purcell swimmer in a round swimming pool — 339
- 10.6 Conclusions and future work — 341

Zheng Chen and Yacine Chitour

Controllability of Keplerian motion with low-thrust control systems — 344

- 11.1 Introduction — 344
- 11.2 Notations and definitions — 345
 - 11.2.1 Dynamics — 345
 - 11.2.2 Study of the drift vector field in $\mathcal{X}$ — 347
 - 11.2.3 Admissible controlled trajectory of Σ_{sat} — 349
 - 11.2.4 Controlled problems in $\mathcal{A}$ — 350
- 11.3 Controllability — 351
 - 11.3.1 Controllability for OTP — 353
 - 11.3.2 Controllability for OIP — 355
 - 11.3.3 Controllability for DOP — 357
- 11.4 Numerical examples — 357
 - 11.4.1 A numerical example for OIP — 357
 - 11.4.2 A numerical example for DOP — 359

- 11.5 Conclusion — **362**
- 11.6 Appendix — **362**

Thierry Combet

Higher variational equation techniques for the integrability of homogeneous potentials — 365

- 12.1 Introduction: integrable systems — **365**
- 12.2 An algebraic point of view — **366**
 - 12.2.1 Algebraic presentation of a Hamiltonian system — **366**
 - 12.2.2 First-order variational equations — **371**
 - 12.2.3 Differential Galois theory — **374**
- 12.3 Introduction to Morales–Ramis theorem — **376**
 - 12.3.1 The Morales–Ramis theorem — **376**
 - 12.3.2 Homogeneous potentials — **377**
 - 12.3.3 Higher variational equations — **378**
- 12.4 Application to parametrized potentials — **381**
 - 12.4.1 Space of germs of integrable potentials — **381**
 - 12.4.2 Eigenvalue bounding of some n -body problems — **384**

Jacques Féjoz

Introduction to KAM theory with a view to celestial mechanics — 387

- 13.1 Twisted conjugacy normal form — **388**
- 13.2 One step of the Newton algorithm — **391**
- 13.3 Inverse function theorem — **396**
- 13.4 Local uniqueness and regularity of the normal form — **399**
- 13.5 Conditional conjugacy — **403**
- 13.6 Invariant torus with prescribed frequency — **404**
- 13.7 Invariant tori with unprescribed frequencies — **408**
- 13.8 Symmetries — **411**
- 13.9 Lower dimensional tori — **413**
- 13.10 Example in the spatial three-body problem — **415**
- A Isotropy of invariant tori — **423**
- B Two basic estimates — **424**
- C Interpolation of spaces of analytic functions — **426**

Marek Grochowski and Wojciech Kryński

Invariants of contact sub-pseudo-Riemannian structures and Einstein–Weyl geometry — 434

- 14.1 Introduction — **434**
- 14.2 Dimension 3 — **435**
- 14.3 Einstein–Weyl geometry — **440**
- 14.4 Dimension $2n + 1$ — **444**

- 14.5 Contact sub-pseudo-Riemannian symmetries — 447
- 14.6 Appendix: Isometries in dimension 5 — 450

Jérôme Lohéac and Jean-François Scheid

Time-optimal control for a perturbed Brockett integrator — 454

- 15.1 Introduction — 454
- 15.2 Controllability and time-optimal controllability — 456
- 15.3 An approximate linearized time-optimal control problem — 457
- 15.4 Numerical computation of a time-optimal trajectory — 459
 - 15.4.1 Finite-dimensional minimization problem — 459
 - 15.4.2 Numerical example — 460
- 15.5 Application to time-optimal micro-swimmers — 463
 - 15.5.1 Modeling and problem formulation — 463
 - 15.5.2 Numerical computation of a time-optimal trajectory — 468
- 15.6 Conclusion — 470

Jean-Pierre Marco

Twist maps and Arnold diffusion for diffeomorphisms — 473

- 16.1 From Arnold diffusion to twist maps — 473
- 16.2 Setting and main result — 479
- 16.3 Proof of Theorem 1 — 483
 - 16.3.1 Choice of $\varepsilon > 0$ — 484
 - 16.3.2 Implementation of Moeckel's method — 488
 - 16.3.3 Normally hyperbolic shadowing — 491
 - 16.3.4 Conclusion of the proof of Theorem 1 — 494

Gianna Stefani and Pierluigi Zezza

A Hamiltonian approach to sufficiency in optimal control with minimal regularity conditions: Part I — 496

- 17.1 Introduction — 496
 - 17.1.1 The problem — 498
- 17.2 Notations and preliminary results — 499
 - 17.2.1 Symplectic notations — 500
 - 17.2.2 The Pontryagin maximum principle — 501
- 17.3 A Hamiltonian approach to optimality — 503
 - 17.3.1 The Cartan form — 503
 - 17.3.2 The super-Hamiltonian and its properties — 507
 - 17.3.3 Abstract sufficient optimality conditions — 509
 - 17.3.4 The minimum time problem — 512
- 17.4 Final comments — 513

Index — 517