

Ching Shen

Rarefied Gas Dynamics

Fundamentals, Simulations and Micro Flows

With 53 Figures



Springer

CONTENTS

Preface	v
Nomenclature	xi
0 Introduction	1
0.1 The Conception of Rarefied Gas Dynamics	1
0.2 The Molecular Model of Gases	3
0.3 Mean Free Path of Molecules	4
0.4 Division of Flow Regimes	5
0.5 Nonequilibrium Phenomena and Rarefied Gas Dynamics	9
0.6 Similarity Criteria	13
References	18
1 Molecular Structure and Energy States	21
1.1 Diatomic Molecules	21
1.2 Energy Distribution of Molecules	30
1.2.1 Boltzmann's Relation	32
1.2.2 Calculation of The Number Ω of Microscopic States	34
1.2.3 Boltzmann Distribution	37
1.3 Internal Energy Distribution Functions	43
References	50
2 Some Basic Concepts of Kinetic Theory	51
2.1 The Velocity Distribution Function	51
2.2 Macroscopic Properties	53
2.3 Binary Elastic Collisions of Molecules	61
2.4 Collision Cross-sections and Molecule Models	69
2.4.1 Hard Sphere Model	72

2.4.2	The Inverse Power Law Model	74
2.4.3	Maxwell Model.....	76
2.4.4	Variable Hard Sphere (VHS) Model.....	77
2.4.5	Variable Soft Sphere (VSS) Model	80
2.4.6	Generalized Hard Sphere (GHS) Model.....	85
2.4.7	Generalized Soft Sphere (GSS) Model	87
2.5	The Eight Velocity Gas Model.....	88
2.6	Boltzmann Equation.....	92
2.7	Collision Integral and The Total Number of Collisions	98
2.8	Evaluation of Collision Integrals	100
2.9	The Maxwell Transport Equation – The Moment Equation	104
2.10	Maxwell Distribution.....	106
2.11	Equilibrium State of Gases	112
2.11.1	Some Peculiar Speeds of Gas	112
2.11.2	Molecular Collision Frequency and The Mean Free Path	115
2.11.3	The Mean Value of Collision Quantities	120
2.11.4	The Reference Diameter of The VSS Model and The VHS Model.....	123
2.12	Gas Mixture	124
2.12.1	The Macroscopic Properties	124
2.12.2	The Boltzmann Equations.....	126
2.12.3	Number of Collisions, Collision Frequency and Mean Free Path.....	126
2.12.4	Collision Frequency of a Molecule of species A with Molecules of Species B in Gas Mixture of VSS (or VHS) Molecules	127
	References	128
3	Interaction of Molecules with Solid Surface	131
3.1	Introduction.....	131
3.2	Specular and Diffuse Reflection	132
3.3	The Reciprocity Principle	140
3.4	The CLL Gas Surface Interaction Model.....	142
	References	158

4	Free Molecular Flow	159
4.1	The Number Flux and The Momentum Flux of Molecules in Gases	160
4.2	The Aerodynamic Forces Acted on Bodies	163
4.3	Heat Transfer to Surface Element.....	170
4.4	Free Molecular Effusion and Thermal Transpiration.....	174
4.5	Couette Flow and Heat Transfer between Plane Plates	177
4.6	The general solutions, unsteady flow.....	182
	References	190
5	Continuum Models	191
5.1	Introduction.....	191
5.2	Basic Equations.....	192
5.2.1	Equations of Mass, Momentum and Energy Conservation..	192
5.2.2	Chapman-Enskog Expansion.....	193
5.2.3	Euler Equation.....	194
5.2.4	Navier-Stokes Equations	194
5.2.5	Burnett Equations	196
5.2.6	Grad's Thirteen Moment Equations.....	201
5.2.7	The Asymptotic Theory for Small Knudsen Numbers	203
5.3	Slip Boundary Conditions.....	204
5.3.1	The Simple Derivation.....	204
5.3.2	The Conservation of Momentum and Energy Fluxes in The Knudsen Layer	206
5.3.3	The Derivation of The Slip Velocity Formula	207
5.3.4	The Derivation of The Temperature Jump Expression.....	209
5.3.5	The Extension to Cases of Multi-component Gases and Non-equilibrium Flows	212
5.4	The Solution of Some Simple Problems	212
5.4.1	Couette Flow.....	213
5.4.2	The Poiseuille Flow	215
5.4.3	The Rayleigh Problem	218
5.5	Thermal Creep and Thermophoresis	220
5.6	Second Order Slip-jump Conditions.....	227

References	228
6 Transitional Regime	231
6.1 General Overview	231
6.2 Linearized Boltzmann Equation	233
6.3 The Moment Method	239
6.4 Model Equations	247
6.5 The Finite Difference Method	255
6.6 Discrete Ordinate Method	257
6.7 Integral Methods	263
6.8 Direct Simulation Methods	264
References	269
7 Direct Simulation Monte-Carlo (DSMC) Method	275
7.1 Introduction	275
7.2 Sampling of Collisions	278
7.3 Example of Solution of Problem by The DSMC Method	281
7.4 The Excitation and Relaxation of The Internal Energies	288
7.4.1 Introduction of Phenomenological Models	288
7.4.2 Implementation of Larsen-Borgnakke Model	289
7.4.3 Cases of Distributions with Singularities, Generalized Acceptance-rejection Method	293
7.4.4 Larsen- Borgnakke Method for Discrete Energy Levels	295
7.4.5 Relaxation Collision Number and Vibrational Exchange Probability	297
7.5 Simulation of Chemical Reactions	299
7.5.1 Chemical Reaction Rate Coefficient	299
7.5.2 Phenomenological Chemical Reaction Model of Bird	300
7.5.3 A Sterically Dependent Chemical Reaction Model	302
7.6 Computation of Comp licated Flow Fields	310
References	313
8 Microscale Slow Gas Flows , Information Preservation Method	317
8.1 Introduction	317

8.2	Methods for Solving The Rarefied Gas Flows in MEMS	321
8.3	Information Preservation (IP) Method.....	326
8.3.1	The Description of The Method.....	326
8.3.2	The Validation of The Method.....	329
8.3.3	Program Demonstrating The Method.....	332
8.4	Unidirectional Flows.....	333
8.5	The Microchannel Flow Problem.....	338
8.6	Thin Film Air Bearing Problem.....	348
8.7	Use of Degenerated Reynolds Equation in Channel Flow	355
8.8	Some Actual Problems and Concluding Remarks	360
	References	363
Appendix I Gas Properties		367
	References	368
Appendix II Some Integrals		369
II.1	The gamma Function and Error Function	369
II.2	Some Definite Integrals	370
II.3	The beta Function.....	373
	References	374
Appendix III Sampling from a Prescribed Distribution		375
III.1	Inversion of Cumulative Distribution Function	375
III.2	Acceptance-rejection Method.....	378
III.3	Generalized Acceptance-rejection Method.....	378
	References	381
Appendix IV Program of The Couette Flow		383
Subject Index		399