

Contents

Part I Models and Forecast

1 The Concept of Model. What is Remarkable in Mathematical Models	3
1.1 What is Called “Model” and “Modelling”	3
1.2 Science, Scientific Knowledge, Systematisation of Scientific Models	6
1.3 Delusion and Intuition: Rescue via Mathematics	10
1.4 How Many Models for a Single Object Can Exist?	14
1.5 How the Models are Born	16
1.6 Structural Scheme of Mathematical Modelling Procedure	17
1.7 Conclusions from Historical Practice of Modelling: Indicative Destiny of Mechanics Models	19
References	23
2 Two Approaches to Modelling and Forecast	25
2.1 Basic Concepts and Peculiarities of Dynamical Modelling	26
2.1.1 Definition of Dynamical System	26
2.1.2 Non-rigorous Example: Variables and Parameters	28
2.1.3 Phase Space. Conservative and Dissipative Systems. Attractors, Multistability, Basins of Attraction	31
2.1.4 Characteristics of Attractors	35
2.1.5 Parameter Space, Bifurcations, Combined Spaces, Bifurcation Diagrams	40
2.2 Foundations to Claim a Process “Random”	42
2.2.1 Set-Theoretic Approach	42
2.2.2 Signs of Randomness Traditional for Physicists	52
2.2.3 Algorithmic Approach	53
2.2.4 Randomness as Unpredictability	54
2.3 Conception of Partial Determinancy	55
2.4 Lyapunov Exponents and Limits of Predictability	56
2.4.1 Practical Prediction Time Estimator	56

2.4.2	Predictability and Lyapunov Exponent: The Case of Infinitesimal Perturbations	57
2.5	Scale of Consideration Influences Classification of a Process (Complex Deterministic Dynamics Versus Randomness)	61
2.6	“Coin Flip” Example	64
	References	68
3	Dynamical (Deterministic) Models of Evolution	71
3.1	Terminology	71
3.1.1	Operator, Map, Equation, Evolution Operator	71
3.1.2	Functions, Continuous and Discrete time	72
3.1.3	Discrete Map, Iterate	73
3.1.4	Flows and Cascades, Poincare Section and Poincare Map	73
3.1.5	Illustrative Example	73
3.2	Systematisation of Model Equations	75
3.3	Explicit Functional Dependencies	79
3.4	Linearity and Non-linearity	81
3.4.1	Linearity and Non-linearity of Functions and Equations	81
3.4.2	The Nature of Non-linearity	82
3.4.3	Illustration with Pendulums	83
3.5	Models in the form of Ordinary Differential Equations	85
3.5.1	Kinds of Solutions	85
3.5.2	Oscillators, a Popular Class of Model Equations	88
3.5.3	“Standard form” of Ordinary Differential Equations	92
3.6	Models in the Form of Discrete Maps	93
3.6.1	Introduction	93
3.6.2	Exemplary Non-linear Maps	94
3.6.3	Role of Discrete Models	99
3.7	Models of Spatially Extended Systems	105
3.7.1	Coupled Map Lattices	105
3.7.2	Cellular Automata	110
3.7.3	Networks with Complex Topology	112
3.7.4	Delay Differential Equations	113
3.7.5	Partial Differential Equations	114
3.8	Artificial Neural Networks	115
3.8.1	Standard Formal Neuron	116
3.8.2	Architecture and Classification of Neural Networks	118
3.8.3	Basic Properties and Problems	119
3.8.4	Learning	120
	References	121
4	Stochastic Models of Evolution	127
4.1	Elements of the Theory of Random Processes	127
4.1.1	Concept of Random Process	127

4.1.2	Characteristics of Random Process	129
4.1.3	Stationarity and Ergodicity of Random Processes	130
4.1.4	Statistical Estimates of Random Process Characteristics ..	131
4.2	Basic Models of Random Processes	131
4.3	Evolutionary Equations for Probability Distribution Laws	134
4.4	Autoregression and Moving Average Processes	135
4.5	Stochastic Differential Equations and White Noise	138
4.5.1	The Concept of Stochastic Differential Equation	138
4.5.2	Numerical Integration of Stochastic Differential Equations	141
4.5.3	Constructive Role of Noise	143
	References	146

Part II Modelling from Time Series

5	Problem Posing in Modelling from Data Series	151
5.1	Scheme of Model Construction Procedure	151
5.2	Systematisation in Respect of A Priori Information	153
5.3	Specific Features of Empirical Modelling Problems	154
5.3.1	Direct and Inverse Problems	154
5.3.2	Well-posed and Ill-posed Problems	155
5.3.3	Ill-conditioned Problems	157
	References	157
6	Data Series as a Source for Modelling	159
6.1	Observable and Model Quantities	159
6.1.1	Observations and Measurements	159
6.1.2	How to Increase or Reduce a Number of Characterising Quantities	163
6.2	Analogue-to-Digital Converters	164
6.3	Time Series	166
6.3.1	Terms	166
6.3.2	Examples	167
6.4	Elements of Time Series Analysis	172
6.4.1	Visual Express Analysis	172
6.4.2	Spectral Analysis (Fourier and Wavelet Transform)	175
6.4.3	Phase of Signal and Empirical Mode Decomposition	187
6.4.4	Stationarity Analysis	191
6.4.5	Interdependence Analysis	193
6.5	Experimental Example	195
	References	197
7	Restoration of Explicit Temporal Dependencies	201
7.1	Parameter Estimation	201

7.1.1	Estimation Techniques	203
7.1.2	Comparison of Techniques	207
7.2	Approximation	212
7.2.1	Problem Formulation and Terms	212
7.2.2	Parameter Estimation	214
7.2.3	Model Size Selection, Overfitting and Ockham's Razor	215
7.2.4	Selecting the Class of Approximating Functions	220
7.3	Model Validation	222
7.3.1	Independence of Residuals	223
7.3.2	Normality of Residuals	223
7.4	Examples of Model Applications	225
7.4.1	Forecast	225
7.4.2	Numerical Differentiation	227
	References	230
8	Model Equations: Parameter Estimation	233
8.1	Parameter Estimators and Their Accuracy	235
8.1.1	Dynamical Noise	235
8.1.2	Measurement Noise	236
8.2	Hidden Variables	239
8.2.1	Measurement Noise	240
8.2.2	Dynamical and Measurement Noise	244
8.3	What One Can Learn from Modelling Successes and Failures	248
8.3.1	An Example from Cell Biology	249
8.3.2	Concluding Remarks	252
	References	252
9	Model Equations: Restoration of Equivalent Characteristics	255
9.1	Restoration Procedure and Peculiarities of the Problem	256
9.1.1	Discrete Maps	256
9.1.2	Ordinary Differential Equations	257
9.1.3	Stochastic Differential Equations	258
9.2	Model Structure Optimisation	260
9.3	Equivalent Characteristics for Two Real-World Oscillators	262
9.3.1	Physiological Oscillator	262
9.3.2	Electronic Oscillator	266
9.4	Specific Choice of Model Structure	268
9.4.1	Systems Under Regular External Driving	268
9.4.2	Time-Delay Systems	270
	References	272
10	Model Equations: "Black Box" Reconstruction	275
10.1	Reconstruction of Phase Orbit	276

10.1.1	Takens' Theorems	277
10.1.2	Practical Reconstruction Algorithms	284
10.2	Multivariable Function Approximation	290
10.2.1	Model Maps	290
10.2.2	Model Differential Equations	299
10.3	Forecast with Various Models	300
10.3.1	Techniques Which Are not Based on Non-linear Dynamics Ideas	300
10.3.2	Iterative, Direct and Combined Predictors	301
10.3.3	Different Kinds of Model Maps	302
10.3.4	Model Maps Versus Model ODEs	303
10.4	Model Validation	304
	References	305
11	Practical Applications of Empirical Modelling	309
11.1	Segmentation of Non-stationary Time Series	310
11.2	Confidential Information Transmission	312
11.3	Other Applications	314
	References	317
12	Identification of Directional Couplings	319
12.1	Granger Causality	319
12.2	Phase Dynamics Modelling	322
12.3	Brain – Limb Couplings in Parkinsonian Resting Tremor	326
12.4	Couplings Between Brain Areas in Epileptic Rats	329
12.5	El Niño – Southern Oscillation and North Atlantic Oscillation	333
12.5.1	Phase Dynamics Modelling	333
12.5.2	Granger Causality Analysis	335
12.6	Causes of Global Warming	337
12.6.1	Univariate Models of the GST Variations	338
12.6.2	GST Models Including Solar Activity	341
12.6.3	GST Models Including Volcanic Activity	343
12.6.4	GST Models Including CO ₂ Concentration	343
	References	344
13	Outdoor Examples	349
13.1	Coupled Electronic Generators	349
13.1.1	Object Description	349
13.1.2	Data Acquisition and Preliminary Processing	351
13.1.3	Selection of the Model Equation Structure	353
13.1.4	Model Fitting, Validation and Usage	355
13.2	Parkinsonian Tremor	363
13.2.1	Object Description	363

- 13.2.2 Data Acquisition and Preliminary Processing 364
 - 13.2.3 Selection of the Model Equation Structure 367
 - 13.2.4 Model Fitting, Validation and Usage 368
 - 13.2.5 Validation of Time Delay Estimation 371
- 13.3 El-Niño/Southern Oscillation and Indian Monsoon 375
 - 13.3.1 Object Description 375
 - 13.3.2 Data Acquisition and Preliminary Processing 376
 - 13.3.3 Selection of the Model Equation Structure 378
 - 13.3.4 Model Fitting, Validation and Usage 379
- 13.4 Conclusions 386
- References 386
-
- Summary and Outlook 389**
-
- List of Mathematical Models 395**
-
- List of Real-World Examples 399**
-
- Index 401**