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Solving Algebraic Computational Problems in Geodesy and Geoinformatics

The Answer to Modern Challenges

With 79 Figures

 Springer

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