Contents

1	Sketch of Lagrangian Formalism						
	1.1	Newton's Equation					
	1.2	Galilear	n Transformations: Principle of Galilean Relativity	8			
	1.3		é and Lorentz Transformations: The Principle of Special				
		Relativi	ity	13			
	1.4		le of Least Action	23			
	1.5		onal Analysis	24			
	1.6	Generalized Coordinates, Coordinate Transformations					
		and Syr	mmetries of an Action	29			
	1.7	Exampl	les of Continuous (Field) Systems	36			
	1.8	·		44			
	1.9	Action	of a Constrained System: Justification of the Recipe	51			
	1.10	Descrip	tion of Constrained System by Singular Action	52			
	1.11	Kinetic	Versus Potential Energy: Forceless Mechanics of Hertz	54			
	1.12	Electron	magnetic Field in Lagrangian Formalism	56			
		1.12.1	Maxwell Equations	56			
		1.12.2	Nonsingular Lagrangian Action of Electrodynamics	59			
		1.12.3	Manifestly Poincaré-Invariant Formulation in Terms				
			of a Singular Lagrangian Action	63			
		1.12.4	Notion of Local (Gauge) Symmetry	65			
		1.12.5	Lorentz Transformations of Three-Dimensional Potential:				
			Role of Gauge Symmetry	68			
		1.12.6	Relativistic Particle on Electromagnetic Background	69			
		1.12.7	Poincaré Transformations of Electric and Magnetic Fields .	72			
		•14	Para Para	77			
2		Hamiltonian Formalism					
	2.1						
		2.1.1	Preliminaries				
		2.1.2	From Lagrangian to Hamiltonian Equations	79			
		2.1.3	Short Prescription for Hamiltonization Procedure,	0.2			
			Physical Interpretation of Hamiltonian	83			

digitalisiert durch

x Contents

		2.1.4				
			Formulation			
	2.2	Poisson Bracket and Symplectic Matrix				
	2.3		Solution to Hamiltonian Equations			
	2.4	5 Conserved Quantities and the Poisson Bracket				
	2.5					
	2.6	Phase Space Transformations and Hamiltonian Equations 96				
	2.7		on of Canonical Transformation			
	2.8	canonical Poisson Bracket				
	2.9					
	2.10		nger Equation as the Hamiltonian System 107			
		2.10.1	Lagrangian Action Associated with the Schrödinger			
			Equation			
		2.10.2	Probability as a Conserved Charge Via the Noether Theorem 111			
			onization Procedure in Terms of First-Order Action Functional 113			
	2.12	Hamilto	onization of a Theory with Higher-Order Derivatives114			
		2.12.1	First-Order Trick			
		2.12.2	Ostrogradsky Method			
3	Can	Canonical Transformations of Two-Dimensional Phase Space 119				
	3.1	Time-In	dependent Canonical Transformations			
		3.1.1	Time-Independent Canonical Transformations			
			and Symplectic Matrix			
		3.1.2	Generating Function			
	3.2	Time-D	ependent Canonical Transformations			
		3.2.1	Canonical Transformations and Symplectic Matrix 123			
		3.2.2	Generating Function			
4	Prop	erties of	Canonical Transformations			
	4.1		ce of the Poisson Bracket (Symplectic Matrix)			
	4.2		simal Canonical Transformations: Hamiltonian			
			nerator of Evolution			
	4.3		ing Function of Canonical Transformation			
		4.3.1	Free Canonical Transformation and Its Function $F(q', p', \tau)$ 136			
		4.3.2	Generating Function $S(q, q', \tau)$			
	4.4		es of Canonical Transformations			
		4.4.1	Evolution as a Canonical Transformation: Invariance			
			of Phase-Space Volume			
		4.4.2	Canonical Transformations in Perturbation Theory			
		4.4.3	Coordinates Adjusted to a Surface			
	4.5		mation Properties of the Hamiltonian Action			
	4.6		y: Equivalent Definitions for Canonical Transformation 145			
	4.7	Hamilton–Jacobi Equation				
	4.8		Functional as a Generating Function of Evolution			
	7.0	Action runctional as a Generating Function of Evolution				

Contents xi

5	Integral Invariants					
	5.1	Poincaré-Cartan Integral Invariant				
		5.1.1 Preliminaries	155			
		5.1.2 Line Integral of a Vector Field, Hamiltonian Action,				
		Poincaré-Cartan and Poincaré Integral Invariants				
		5.1.3 Invariance of the Poincaré–Cartan Integral	159			
	5.2	Universal Integral Invariant of Poincaré	162			
6	Pote	ntial Motion in a Geometric Setting	167			
	6.1	Analysis of Trajectories and the Principle of Maupertuis				
		6.1.1 Trajectory: Separation of Kinematics from Dynamics	168			
		6.1.2 Equations for Trajectory in the Hamiltonian Formulation	170			
		6.1.3 The Principle of Maupertuis for Trajectories	171			
		6.1.4 Lagrangian Action for Trajectories	172			
	6.2	Description of a Potential Motion in Terms of a Pair of Riemann				
		Spaces	174			
	6.3	Some Notions of Riemann Geometry	178			
		6.3.1 Riemann Space				
		6.3.2 Covariant Derivative and Riemann Connection	183			
		6.3.3 Parallel Transport: Notions of Covariance and Coordinate				
		Independence	185			
	6.4	Ç				
		Formal Solution to the Parallel Transport Equation				
	6.5	The Geodesic Line and Its Reparametrization Covariant Equation				
	6.6	Example: A Surface Embedded in Euclidean Space	193			
	6.7	Shortest Line and Geodesic Line: One More Example of a Singular				
		Action				
	6.8	Formal Geometrization of Mechanics	200			
7		sformations, Symmetries and Noether Theorem				
	7.1	The Notion of Invariant Action Functional	203			
	7.2	Coordinate Transformation, Induced Transformation of Functions				
		and Symmetries of an Action				
	7.3	Examples of Invariant Actions, Galileo Group				
	7.4	Poincaré Group, Relativistic Particle				
	7.5	Symmetries of Equations of Motion				
	7.6	Noether Theorem				
	7.7	Infinitesimal Symmetries				
	7.8					
	7.9	Use of Noether Charges for Reduction of the Order of Equations	22.4			
	7.10		224 225			
	7.10	Examples	225			

xii Contents

	7.11	1 Symmetries of Hamiltonian Action		
		7.11.1	Infinitesimal Symmetries Given by Canonical	
			Transformations	228
		7.11.2	Structure of Infinitesimal Symmetry of a General Form	
		7.11.3	· · · · · · · · · · · · · · · · · · ·	
8	Ham		Formalism for Singular Theories	
	8.1	Hamilto	onization of a Singular Theory: The Recipe	
		8.1.1	Two Basic Examples	. 238
		8.1.2	Dirac Procedure	. 242
	8.2	Justification of the Hamiltonization Recipe		. 247
		8.2.1	Configuration-Velocity Space	. 247
		8.2.2	Hamiltonization	
		8.2.3	Comparison with the Dirac Recipe	. 252
	8.3		cation of Constraints	
	8.4	Comme	ent on the Physical Interpretation of a Singular Theory	. 255
	8.5	Theory	with Second-Class Constraints: Dirac Bracket	. 259
	8.6	Exampl	les of Theories with Second-Class Constraints	. 262
		8.6.1	Mechanics with Kinematic Constraints	. 262
		8.6.2	Singular Lagrangian Action Underlying	
			the Schrödinger Equation	264
	8.7	Exampl	es of Theories with First-Class Constraints	. 266
		8.7.1	Electrodynamics	
		8.7.2	Semiclassical Model for Description of Non Relativistic	
			Spin	268
	8.8	Local S	ymmetries and Constraints	. 274
	8.9			
	8.10 Formalism of Extended Lagrangian			. 281
	8.11			. 286
	8.12	Local S	ymmetries of the Initial Lagrangian	. 290
8.13 Conversion of Second-Class Constraints by Deformation		sion of Second-Class Constraints by Deformation		
		of Lagra	angian Local Symmetries	293
		8.13.1	Conversion in a Theory with Hidden $SO(1, 4)$	
			Global Symmetry	296
		8.13.2	Classical Mechanics Subject to Kinematic Constraints	
			as a Gauge Theory	298
		8.13.3	Conversion in Maxwell-Proca Lagrangian for Massive	
			Vector Field	301
D	1.			200
Bib	mogra	aphy		. 303
Ind	lex			. 305
				-