Contents

Part I Stochastic Differential Equations in Infinite Dimensions

1 Partial Differential Equations as Equations in Infinite Dimensions . 3
1.1 The Heat Equation as an Abstract Cauchy Problem 3
1.2 Elements of Semigroup Theory 5
1.3 Commonly Used Function Spaces 10
1.4 The Abstract Cauchy Problem 11
1.5 The Variational Method 15

2 Stochastic Calculus 17
2.1 Hilbert-Space-Valued Process, Martingales, and Cylindrical Wiener Process 17
2.1.1 Cylindrical and Hilbert-Space-Valued Gaussian Random Variables 17
2.1.2 Cylindrical and Q-Wiener Processes 19
2.1.3 Martingales in a Hilbert Space 21
2.2 Stochastic Integral with Respect to a Wiener Process 23
2.2.1 Elementary Processes 25
2.2.2 Stochastic Itô Integral for Elementary Processes 26
2.2.3 Stochastic Itô Integral with Respect to a Q-Wiener Process 34
2.2.4 Stochastic Itô Integral with Respect to Cylindrical Wiener Process 44
2.2.5 The Martingale Representation Theorem 49
2.2.6 Stochastic Fubini Theorem 57
2.3 The Itô Formula 61
2.3.1 The Case of a Q-Wiener Process 61
2.3.2 The Case of a Cylindrical Wiener Process 69

3 Stochastic Differential Equations 73
3.1 Stochastic Differential Equations and Their Solutions 73
3.2 Solutions Under Lipschitz Conditions 84
3.3 A Special Case 103

Bibliografische Informationen
http://d-nb.info/1006508031

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

3.4 Markov Property and Uniqueness . . . 107
3.5 Dependence of the Solution on the Initial Value . . . 111
3.6 Kolmogorov's Backward Equation . . . 117
3.7 Lipschitz-Type Approximation of Continuous Coefficients . . . 125
3.8 Existence of Weak Solutions Under Continuity Assumption . . . 128
3.9 Compact Semigroups and Existence of Martingale Solutions . . . 135
3.10 Mild Solutions to SSDEs Driven by Cylindrical Wiener Process . . 141
Appendix: Compactness and Tightness of Measures in $C([0,T],\mathscr{M})$ . . . 148

4 Solutions by Variational Method . . . 151
4.1 Introduction . . . 151
4.2 Existence of Weak Solutions Under Compact Embedding . . . 152
4.3 Strong Variational Solutions . . . 174
4.4 Markov and Strong Markov Properties . . . 181

5 Stochastic Differential Equations with Discontinuous Drift . . . 185
5.1 Introduction . . . 185
5.2 Unbounded Spin Systems, Solutions in $C([0,T],H^w)$ . . . 185
5.3 Locally Interacting Particle Systems, Solutions in $C([0,T],\mathbb{R}^{\mathbb{Z}^d})$ . . . 194

Part II Stability, Boundedness, and Invariant Measures

6 Stability Theory for Strong and Mild Solutions . . . 203
6.1 Introduction . . . 203
6.2 Exponential Stability for Stochastic Differential Equations . . . 211
6.3 Stability in the Variational Method . . . 225
Appendix: Stochastic Analogue of the Datko Theorem . . . 230

7 Ultimate Boundedness and Invariant Measure . . . 233
7.1 Exponential Ultimate Boundedness in the m.s.s. . . . 233
7.2 Exponential Ultimate Boundedness in Variational Method . . . 237
7.3 Abstract Cauchy Problem, Stability and Exponential Ultimate Boundedness . . . 246
7.4 Ultimate Boundedness and Invariant Measure . . . 255
7.4.1 Variational Equations . . . 261
7.4.2 Semilinear Equations Driven by a Q-Wiener Process . . . 266
7.4.3 Semilinear Equations Driven by a Cylindrical Wiener Process . . . 271
7.5 Ultimate Boundedness and Weak Recurrence of the Solutions . . . 274

References . . . 285

Index . . . 289