

Contents

Part I

Introduction.....	1
1 The Generalized Class Number Formulas	7
1.1 Abelian Number Fields as Class Fields.....	7
1.2 The Analytic Class Number Formula.....	9
1.3 Product Formulas for the Conductors and for the Gaussian Sums.....	11
1.4 Calculation of L -series	12
1.5 The Arithmetic Class Number Formula	15
1.6 Preliminary Remarks on the Arithmetic Structure of the Two Class Number Factors	17
2 The Arithmetic Structure of the Class Number Formula for Real Fields	23
2.1 Plan of Investigation	23
2.2 The First Way of Transformation	25
2.3 The Number Factor g_K	28
2.4 Introduction to Cyclotomic Units	30
2.5 The First Arithmetic Representation of the Class Number	33
2.6 The Theorem of Weber and Its Generalization	37
2.7 Generalized Group Matrix	42
2.8 Linear Factor Decomposition of Generalized Group Determinant	45
2.9 The Number Factor $c_{\mathfrak{G}}$	48
2.10 The Second Way of Transformation	52
2.11 The Second Arithmetic Representation of the Class Number	54
2.12 Real Cyclic Biquadratic Fields.....	57

3 The Arithmetic Structure of the Relative Class Number	
Formula for Imaginary Fields	65
3.1 Class-Field-Theoretic Proof of the Rational Integrality and the Arithmetic Meaning	65
3.2 The Unit Index Q	73
3.3 Criterion for $Q = 1$ or 2 by a Kummer-Generator	77
3.4 Criterion for $Q = 1$ or 2 by Ramification and Class Problem	81
3.5 Description of Ramification by the Characters	85
3.6 Criteria for $Q = 1$ or 2 by Characters and Class Problem	92
3.7 Types of Fields with $Q = 1$ and Types of Fields with $Q = 2$	94
3.8 Imaginary Bicyclic Biquadratic Fields	100
3.9 Preparation for Direct Proof of the Integrality	108
3.10 The Characters with Composite Conductor	111
3.11 Supplement of Gauss' Lemma	116
3.12 The Characters of 2-Power Order and with Composite Conductor	121
3.13 The Characters of Odd Prime-Power Conductor	125
3.14 The Characters of 2-Power Conductor	130
3.15 Direct Proof of the Integrality	133
3.16 Theorem of Weber and Its Supplement	140
3.17 Remarks on the Genus Factor	161
3.18 Divisibility by the Relative Class Number of a Subfield	162
3.19 Imaginary Abelian Number Fields with Odd Class Number	167
3.20 Imaginary Cyclic Fields with Odd Class Number	175
Appendix: Tables of Relative Class Numbers	188
Table of Contributions to Relative Class Number	188
Table of Relative Class Numbers	191
 Part II	
4 On the Relative Class Number of the Imaginary Abelian Number Field I	255
5 On the Relative Class Number of the Imaginary Abelian Number Field II	307
6 Supplemental Readings	359
Bibliography on Hasse's Unit Indices of CM-Fields	359
Bibliography on Calculations of Relative Class Numbers of Imaginary Abelian Number Fields	360
Bibliography on Determinantal Expressions of Relative Class Numbers of Imaginary Abelian Number Fields	361
Index	365