

Contents

Preface — VII

List of symbols — XIII

1	Sets, Relations and Functions — 1
1.1	Sets and subsets — 1
1.2	Relations, equivalence relations, partial orders — 5
1.3	Functions — 10
1.4	Cardinality — 14
1.5	Zorn's Lemma and variants — 18
2	The Integers — 20
2.1	Well-ordering and mathematical induction — 20
2.2	Division in the integers — 22
2.3	Congruences — 28
3	Introduction to Groups — 35
3.1	Permutations — 35
3.2	Semigroups, monoids and groups — 44
3.3	Groups and subgroups — 49
4	Quotient groups and Homomorphisms — 58
4.1	Cosets and Lagrange's Theorem — 58
4.2	Normal subgroups and quotient groups — 67
4.3	Homomorphisms — 75
5	Groups Acting on Sets — 86
5.1	Group actions — 86
5.2	Orbits and stabilizers — 89
5.3	Applications to the structure of groups — 93
6	Introduction to rings — 102
6.1	Elementary properties of rings — 102
6.2	Subrings and ideals — 107
6.3	Integral domains, division rings and fields — 114
6.4	Finiteness conditions on ideals — 121
7	Division in Commutative Rings — 125
7.1	Euclidean domains — 125
7.2	Principal ideal domains — 128

7.3	Unique factorization in integral domains —	131
7.4	Roots of polynomials and splitting fields —	137
8	Vector Spaces —	144
8.1	Vector spaces and subspaces —	144
8.2	Linear independence, basis and dimension —	148
8.3	Linear mappings —	158
8.4	Eigenvalues and eigenvectors —	167
9	Introduction to Modules —	180
9.1	Elements of module theory —	180
9.2	Modules over principal ideal domains —	197
9.3	Applications to linear operators —	210
10	The Structure of Groups —	220
10.1	The Jordan–Hölder Theorem —	220
10.2	Solvable and nilpotent groups —	229
10.3	Theorems on finite solvable groups —	236
11	The Theory of Fields —	243
11.1	Field extensions —	243
11.2	Constructions with ruler and compass —	251
11.3	Finite fields —	256
12	Galois Theory —	261
12.1	Normal and separable extensions —	261
12.2	Automorphisms of fields —	266
12.3	The Fundamental Theorem of Galois theory —	274
12.4	Solvability of equations by radicals —	280
12.5	Roots of Polynomials and Discriminants —	288
13	Tensor Products —	292
13.1	Definition of the tensor product —	292
13.2	Properties of tensor products —	298
13.3	Extending the ring of operators —	305
14	Representations of groups —	309
14.1	Representations and group rings —	309
14.2	The structure of group algebras —	314
14.3	Group characters —	318
14.4	The Burnside $p - q$ Theorem —	329

15	Presentations of groups — 332
15.1	Free groups — 332
15.2	Generators and relations — 340
15.3	Free products — 345
16	Introduction to category theory — 354
16.1	Categories — 354
16.2	Functors — 357
16.3	Categorical constructions — 366
16.4	Natural transformations — 375
17	Applications — 380
17.1	Set labelling problems — 380
17.2	Enumerating graphs — 384
17.3	Latin squares and Steiner systems — 386
17.4	Introduction to error correcting codes — 395
17.5	Algebraic models for accounting systems — 408
Bibliography — 429	
Index — 431	